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CLASS XI & XII MATHEMATICS

UNIT 9: LINEAR EQUATIONS & FEASIBLE REGIONS

LINEAR INEQUALITIES

Comprehensive Study Notes, Practical Case Studies & DPPs

With Kumar's Hand-Written Style Classroom Notes & Question Paper

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Prepared by:	Kumar Classes Mathematics Department

1 Module 1: Linear Inequalities in One Variable

In real-world scenarios, mathematical equations do not always represent strict equality. Often, limits and boundaries define are defined as inequalities. This module focuses on the properties, transposition rules, and analytical solution techniques of linear inequalities in one variable.

Core Mathematical Definitions

An inequality is of the form $ax + b < c$, $ax + b > c$, $ax + b \leq c$, or $ax + b \geq c$, where a, b, c are real numbers, $a \neq 0$, and x is a variable. These are called **linear inequations in one variable** x [1, 2].

- **Replacement Set (Domain):** The set of values from which x can be chosen is called the replacement set or the domain of the variable [2].
- **Solution Set:** The set of all values from which x satisfies the given inequality is called the solution set [3]. The solution set of a linear inequality can be a finite set or an infinite set depending on the domain [3].
- **Transposition:** A term can be shifted from one side of an inequation to the other side with its sign reversed without affecting the inequality [2].

1.1 Fundamental Properties of Inequalities

To solve inequalities without altering the truth value of the expression, we must adhere strictly to three foundational algebraic properties [4]:

I. Additive Property: Adding or subtracting the same number to each side of an inequation does not change the direction of the inequality [4]:

$$\text{If } b < c, \text{ then } b + a < c + a \text{ and } b - a < c - a$$

II. Multiplicative Property (Positive multiplier): Multiplying or dividing each side of an inequation by the same positive number does not change the direction of the inequality [4]:

$$\text{If } b < c \text{ and } a > 0, \text{ then } ba < ca \text{ and } \frac{b}{a} < \frac{c}{a}$$

III. Multiplicative Property (Negative multiplier – CRITICAL): Multiplying or dividing each side of an inequation by the same negative number **REVERSES** the direction of the inequality [4]:

$$\text{If } b < c \text{ and } a < 0, \text{ then } ba > ca \text{ and } \frac{b}{a} > \frac{c}{a}$$

Kumar's Classroom Warning: Negatives and Inequality Signs

Never forget: multiplying or dividing an inequality by a negative number flips the sign. For instance, solving $-3x \leq 12$ requires dividing by -3. Doing so gives $x \geq -4$, NOT $x \leq -4$. Mistakes with negative signs are the most common source of error in school board exams!

1.2 Standard Interval Notation

Solutions to inequalities on the real number line ($\mathbb{R}$) are represented using interval notation:

- **Closed Interval** $[a, b]$**:** $\{x \in \mathbb{R} : a \leq x \leq b\}$. Both boundary points are included [11].
- **Open Interval** (a, b)**:** $\{x \in \mathbb{R} : a < x < b\}$. Boundary points are excluded.
- **Semi-open Interval** $[a, b)$**:** $\{x \in \mathbb{R} : a \leq x < b\}$. a is included, b is excluded [12].

1.3 Rigorous Algebraic Worked Examples

Let's study three core exam-standard algebraic inequality formats under different domains.

Kumar's Solved Example: Example 1.1 – Integer Domain restriction

Solve the inequality for $x \in \mathbb{I}$ (Integers) [8]:

$$4(x+5) > -10$$

Solution. We start with the given expression and expand the brackets [9]:

$$\begin{aligned} 4x + 20 &> -10 \\ 4x &> -10 - 20 \quad \text{(by Transposition) [9]} \\ 4x &> -30 \\ x &> \frac{-30}{4} \\ x &> -7.5 \end{aligned}$$

Since our domain is limited to Integers ($x \in \mathbb{I}$), the solution set includes all integers strictly greater than -7.5 [9]:

$$\text{Solution Set} = \{-7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, \ldots\} \quad [9]$$

□

Kumar's Solved Example: Example 1.2 – Rational Algebraic Form

Solve the rational inequation for $x \in \mathbb{R}$ [12]:

$$\frac{x+4}{x-2} > 0$$

Solution. For a rational fraction $\frac{A}{B} > 0$, the signs of the numerator and denominator must be identical. This gives us two mutually exclusive cases [12]:

Case I: Both Numerator and Denominator are strictly positive [12]:

$$x + 4 > 0 \quad \text{and} \quad x - 2 > 0 \implies x > -4 \quad \text{and} \quad x > 2[12]$$

The intersection of these two conditions is:

$$x > 2 \implies \text{Interval } (2, \infty) \quad [12]$$

Case II: Both Numerator and Denominator are strictly negative [12]:

$$x + 4 < 0 \quad \text{and} \quad x - 2 < 0 \implies x < -4 \quad \text{and} \quad x < 2[13]$$

The intersection of these two conditions is:

$$x < -4 \implies \text{Interval } (-\infty, -4) \quad [13]$$

Combining both cases, the complete solution set is the union of the two intervals [13]:

$$\text{Solution Set} = (-\infty, -4) \cup (2, \infty) \quad [13]$$

□

Kumar's Solved Example: Example 1.3 – Absolute Value Inequality

Solve the absolute value inequality for $x \in \mathbb{R}$ [16]:

$$|x+2| > 5$$

Solution. An absolute value inequality $|u| > d$ split into two distinct inequalities: $u > d$ or $u < -d$.

Case I: When $x + 2 \geq 0 \implies x \geq -2$ **[16].** The inequality becomes:

$$x + 2 > 5 \implies x > 3 \implies \text{Interval } (3, \infty) \quad [16]$$

Case II: When $x + 2 < 0 \implies x < -2$ **[16].** The inequality becomes:

$$-(x+2) > 5 \implies -x - 2 > 5 \implies -x > 7 \implies x < -7 \quad [16]$$

This represents the interval $(-\infty, -7)$ [16].

Combining both intervals, we get the complete solution set [16]:

$$\text{Solution Set} = (-\infty, -7) \cup (3, \infty) \quad [16]$$

□

2 Module 2: Graphical Solutions of Inequalities in Two Variables

When an inequality contains two variables, its solution set can no longer be represented as simple points on a 1D number line. Instead, it defines a region (a half-plane) in a 2D Cartesian coordinate system [20].

2D Graphical Boundary Logic

An equation of the form $ax + by = c$ represents a straight line that divides the Cartesian plane into two distinct half-planes [18, 20]. An inequality of the form $ax + by < c$ or $ax + by > c$ is represented graphically by one of these half-planes [19, 20].

- **Strict Inequality ($<$ or $>$):** The boundary line $ax + by = c$ is drawn as a **dashed (dotted) line**, indicating that points lying on the line itself are excluded from the solution set [31].
- **Slack Inequality ($\leq$ or $\geq$):** The boundary line is drawn as a **solid line**, indicating that points lying on the line are included [27].
- **The Origin Test:** If a line does not pass through the origin ($c \neq 0$), we can use the origin $(0, 0)$ as a test point to determine which half-plane contains the solution set [20]. If substituting $(0, 0)$ yields a true statement, the solution set is the half-plane containing the origin [21]. If false, it is the non-origin side [21].

2.1 Rigorous Two-Variable Graphical Example

Let's walk through the exact steps required to solve a system of linear inequalities graphically.

Kumar's Solved Example: Example 2.1 – Two-Variable Graphical System

Solve the following system of linear inequalities graphically and find the coordinates of the vertices of the common region [33]:

$$2x + y \geq 2 \quad \textit{and} \quad x - y \leq 1$$

Solution. We analyze each inequality step-by-step:
Inequality 1: $2x + y \geq 2$ **[33]**

1. Find the intercept points for the boundary line $2x + y = 2$ [33]:
   - Put $y = 0 \implies 2x = 2 \implies x = 1$. Intercept point is $A(1, 0)$ [33].
   - Put $x = 0 \implies y = 2$. Intercept point is $B(0, 2)$ [33].
2. Perform the Origin Test [33]: Substitute $(0, 0)$ into $2(0) + 0 \geq 2 \implies 0 \geq 2$. This statement is **FALSE** [33].
3. Therefore, the solution is the **non-origin side** of the solid line $2x + y = 2$ [33].

Inequality 2: $x - y \leq 1$ **[33]**

1. Find the intercept points for the boundary line $x - y = 1$ [33]:
   - Put $y = 0 \implies x = 1$. Intercept point is $A(1, 0)$ [33].
   - Put $x = 0 \implies -y = 1 \implies y = -1$. Intercept point is $C(0, -1)$ [33].

2. Perform the Origin Test [33]: Substitute $(0, 0)$ into $0 - 0 \leq 1 \implies 0 \leq 1$. This statement is **TRUE** [33].

3. Therefore, the solution is the **origin side** of the solid line $x - y = 1$ [33].

The common region is the intersection of these two half-planes. Since the intersection point of the two boundary lines occurs at $A(1, 0)$, this is the primary vertex of the shaded region. □

3 Module 3: Convex Sets Linear Programming Formulations

A key application of linear inequalities is Linear Programming (LPP), which is used extensively in operations research, business budgeting, and resource optimization.

Convex Sets and Feasible Solutions

- **Convex Set:** A set of points in a plane is convex if the line segment joining any two points in the set lies entirely within the set [36].
- **Polygonal Convex Set:** If the boundaries of a convex set are formed by straight lines, it is called a polygonal convex set [37].
- **Convex Polygon:** A bounded polygonal convex set is called a convex polygon [37].
- **Feasible Solution:** The coordinates of any point in the plane that satisfies all the inequalities of a given system simultaneously is called a feasible solution [37].

3.1 Standard LPP Case Studies

Let's study the formulation of three standard corporate resource allocation problems from the source material.

LPP Case Study 1: Gold & Silver Jewelry Production

Problem Scenario: A company produces two types of articles, A and B, which require gold and silver. Each unit of A requires 3g of silver and 1g of gold. Each unit of B requires 2g of silver and 2g of gold. The company has at most 6g of silver and 4g of gold available [45].
Mathematical Formulation [46]: Let the company produce x units of article A and y units of article B [46]. Since we cannot produce negative units, we have the non-negativity constraints:

$$x \geq 0, \quad y \geq 0 \quad [46]$$

Now, we map the resource limitations to linear inequalities [46]:

$$\textbf{Silver Constraint:} \quad 3x + 2y \leq 6 \quad [46]$$
$$\textbf{Gold Constraint:} \quad x + 2y \leq 4 \quad [46]$$

The feasible solution region is the bounded convex polygon $OABC$ in the first quadrant, with vertices at $O(0, 0)$, $A(2, 0)$, $B(1, 1.5)$, and $C(0, 2)$.

LPP Case Study 2: Furniture Dealer Inventory Optimization

Problem Scenario: A furniture dealer deals in tables and chairs. He has Rs. 15,000 to invest and a storage space that can hold at most 60 pieces. A table costs him Rs. 150, and a chair costs him Rs. 750 [47].
Mathematical Formulation [47]: Let x be the number of tables and y be the number of chairs.

$$x \geq 0, \quad y \geq 0 \quad [48]$$

$$\textbf{Storage Space Constraint:} \quad x + y \leq 60 \quad [47]$$
$$\textbf{Investment Budget Constraint:} \quad 150x + 750y \leq 15000 \implies x + 5y \leq 100 \quad [47]$$

The shaded region $OABC$ represents the feasible region, where the vertices are $O(0,0)$, $A(60,0)$, $B(50,10)$, and $C(0,20)$ [48].

4 Daily Practice Problems (DPP) Series

Each DPP contains questions designed to test your understanding of the concepts covered in this unit. Detailed solutions are provided at the end of this booklet.

4.1 DPP Set 1: One-Variable Algebraic Absolute Value Inequalities

1. If $x \in \{-3, -2, -1, 0, 1, 2, 3\}$, solve the inequality: $2x + 6 > 5$ and represent the solution set on a number line [5, 6].
2. If $x \in \mathbb{I}$ (Integers), solve: $9x - 25 \leq 7 + 3x$ [8].
3. If $x \in \mathbb{I}$, solve: $10 - 2(1 - 4x) < 20$ [10].
4. Solve for $x \in \mathbb{R}$: $-3 \leq 3 - 2x \leq 9$ [11].
5. Solve for $x \in \mathbb{R}$: $\frac{x-3}{x+4} < 0$ and represent it on a number line [12, 13].
6. Solve the rational inequality for $x \in \mathbb{R}$: $\frac{2x-4}{x-1} \geq 1$ [50].
7. Solve the absolute value inequality: $2|4 - 5x| \geq 9$ [84].
8. If $x \in \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$, solve the inequality: $\frac{3}{2} \leq 1 - \frac{x}{2}$ [79, 80].

4.2 DPP Set 2: Graphical Solutions of Systems of Inequalities

Solve the following systems of inequalities graphically and locate all corner vertices of the region:

1. $x \geq 2$, $y \geq 3$, and $x + y \leq 6$.
2. $2x + 3y \leq 6$, $x + y \geq 2$, $x \geq 0$, $y \geq 0$ [39].
3. $3x + 4y \leq 12$, $4x + 7y \leq 28$, $y \geq 1$, $x \geq 0$, $y \geq 0$ [39].
4. $x + 2y \leq 100$, $2x + y \leq 120$, $x + y \geq 70$, $x \geq 0$, $y \geq 0$ [41].
5. $x + 4y \leq 24$, $3x + y \leq 21$, $x + y \geq 9$, $x \geq 0$, $y \geq 0$ [42].
6. $0 \leq x \leq 3$, $0 \leq y \leq 3$, $x + y \geq 5$, $2x + y \geq 4$ [43].
7. $x - 2y \leq 2$, $x + y \geq 3$, $-2x + y \leq 4$, $x \geq 0$, $y \geq 0$ [44].
8. $12x + 15y \leq 180$, $3x - 4y \leq 12$, $x \geq 0$, $y \geq 0$ [60].

4.3 DPP Set 3: LPP Formulations Feasible Region Modeling

Formulate the following word problems as systems of linear inequalities and sketch their feasible regions:

1. **Production Optimization:** A factory uses three different resources to manufacture two products. The maximum availabilities are 20 units of resource A, 12 units of resource B, and 16 units of resource C. One unit of the first product requires 2 units of A, 2 units of B, and 4 units of C. One unit of the second product requires 4 units of A, 2 units of B, and no units of C [48].
2. **Transportation Logistics:** A dealer can sell two types of cloth, A and B. He can sell at most 400 units of A and 300 units of B. He cannot have more than 600 units of A and B together. Formulate the constraints and map the feasible region [70, 71].

3. **Nutritional Optimization:** A person requires 10, 12, and 12 units of chemicals A, B, and C respectively. A liquid product contains 5, 2, and 1 units of these chemicals per jar. A dry product contains 1, 2, and 4 units of these chemicals per carton [72]. Formulate the minimum requirements [72, 73].

4. **Industrial Manufacturing:** A firm manufactures products A and B processed on two machines M_1 and M_2. Product A requires 1 minute on M_1 and 2 minutes on M_2. Product B requires 1 minute on M_1 and 1 minute on M_2. Machine M_1 is available for at most 450 minutes, and machine M_2 is available for at most 600 minutes per day [74]. Formulate the LPP [75].