KUMAR CLASSES
KUMARNETIC PVT. LTD. · Inspire · Innovate · Excel

ICSE · Class X
Mathematics
Sample Paper — Set 1

MATHEMATICS — ICSE BOARD PATTERN

Class X · Sample / Preboard Paper · **SET 1 of 3** · GST · Banking · Inequations · Quadratics · Proportion · Factorisation · Matrices · A.P. · Similarity · Statistics

Maximum Marks : 80 | **Time : Two & a half Hours (2½ h)** | **Reading time : 15 min**

Name : _____ Roll No : _____ Date : _____ Marks : _____ / 80

General Instructions

1. You will **not** be allowed to write during the first 15 minutes. This time is to be spent reading the question paper.
2. The time given at the head of this paper is the time allowed for **writing** the answers.
3. **Attempt all questions from Section A** and **any four questions from Section B**.
4. All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks.
5. The intended marks for questions or parts of questions are given in brackets **[ ]**.
6. Mathematical tables and graph paper are provided.

SECTION A — 40 Marks

(Attempt ALL questions from this Section.)

Q1. Choose the correct answers to the questions from the given options. **[15]**

(i) The amount of GST on an article of marked price ₹2,000 at the rate 12% is:

(a) ₹120 (b) ₹200
(c) ₹240 (d) ₹180

(ii) In a recurring deposit of ₹P per month for n months, the equivalent principal for one month's interest is:

(a) $P \cdot n$ (b) $P \cdot \frac{n(n-1)}{2}$
(c) $P \cdot \frac{n^2}{2}$ (d) $P \cdot \frac{n(n+1)}{2}$

(iii) The solution set of $2x - 3 \leq 7$, $x \in N$ is:

(a) {1,2,3,4} (b) {1,2,3,4,5}

(c) {0,1,2,3,4,5} (d) {1,2,3,4,5,6}

(iv) The roots of the equation $x^2 - 4x + 4 = 0$ are:

(a) real and unequal (b) real and equal

(c) imaginary (d) not real

(v) If $x = 2$ is a root of $x^2 - 5x + k = 0$, then $k =$

(a) 6 (b) −6

(c) 4 (d) −4

(vi) The mean proportional between 9 and 16 is:

(a) 7 (b) 25

(c) 12 (d) 144

(vii) The remainder when $x^3 - 2x^2 + 3x - 1$ is divided by $(x - 1)$ is:

(a) 0 (b) 1

(c) −1 (d) 5

(viii) $(x - 1)$ is a factor of $x^2 - 3x + k$ if $k =$

(a) 2 (b) −2

(c) 4 (d) 1

(ix) If A is a 2×3 matrix and B is a 3×2 matrix, then the order of AB is:

(a) 3×3 (b) 2×2

(c) 2×3 (d) not defined

(x) If $A = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$, the element in the first row and first column of 2A is:

(a) 1 (b) 3

(c) 2 (d) 8

(xi) The 10th term of the A.P. 2, 5, 8, ... is:

(a) 27 (b) 32

(c) 29 (d) 30

(xii) For an A.P. whose nth term is $t_n = 3n + 1$, the common difference is:

(a) 1 (b) 3

(c) 4 (d) 2

(xiii) The areas of two similar triangles whose corresponding sides are in the ratio 3 : 5 are in the ratio:

(a) 3 : 5 (b) 5 : 3

(c) 9 : 25 (d) 27 : 125

(xiv) In $\triangle ABC$, $DE \parallel BC$ with D on AB and E on AC. If $AD/DB = 2/3$, then $AE/EC =$

(a) $\frac{3}{2}$ (b) $\frac{2}{5}$

(c) $\frac{2}{3}$ (d) $\frac{4}{9}$

(xv) For the classes 0–10, 10–20, 20–30 with frequencies 5, 12, 8, the modal class is:

(a) 0–10 (b) 10–20

(c) 20–30 (d) none

Q2. [12]

(i) A dealer in Delhi buys goods worth ₹15,000 from a manufacturer in Delhi and sells them to a consumer in Delhi at a profit of 20%. If the rate of GST is 12%, find (a) the selling price of the goods, and (b) the CGST, SGST and the ***total amount*** paid by the consumer. [4]

(ii) Deepa deposits ₹800 per month in a recurring deposit account for 2 years at 10% per annum. Find (a) the interest earned, and (b) the maturity value of the account. [4]

(iii) If $A = \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}$ and $B = \begin{vmatrix} 2 & -2 \\ 1 & 4 \end{vmatrix}$, find the product AB. [4]

Q3. [13]

(i) Solve the following inequation, write the solution set and represent it on a number line: $-8 < 3x - 2 \leq 7, \quad x \in R$. [4]

(ii) Which term of the A.P. 5, 9, 13, ... is 81? Hence find the sum of the first 15 terms of the A.P. [4]

(iii) The sum of the squares of two consecutive natural numbers is 313. Form a quadratic equation and hence find the numbers. [5]

SECTION B — 40 Marks

(Attempt any FOUR questions from this Section.)

Q4. [10]

(i) Solve the quadratic equation $3x^2 - 5x - 4 = 0$ and give your answer correct to two decimal places. [3]

(ii) What number must be added to each of 6, 15, 20 and 43 so that the resulting numbers are in proportion? [3]

(iii) Using the properties of proportion, solve for x: $\frac{x^3 + 3x}{3x^2 + 1} = \frac{341}{91}$. [4]

Q5. [10]

(i) Find the remainder when $2x^3 + 3x^2 - 5x + 7$ is divided by $(2x - 1)$. [3]

(ii) Using the factor theorem, factorise completely: $x^3 - 7x + 6$. [4]

(iii) If $A = \begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix}$ and $B = \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix}$, find $A - 2B$. **[3]**

Q6. **[10]**

(i) A shopkeeper buys an article whose printed price is ₹8,000 from a wholesaler at a discount of 25% and pays GST at 18%. He sells the article to a consumer at the printed price and charges GST at 18%. Find (a) the price paid by the shopkeeper inclusive of GST, (b) the amount of GST paid by the shopkeeper to the Central Government (CGST), and (c) the amount paid by the consumer for the article. **[6]**

(ii) Mr. Sharma deposits ₹2,500 per month in a recurring deposit account for 1 year. If he receives ₹31,625 at maturity, find the rate of interest per annum. **[4]**

Q7. **[10]**

(i) In $\triangle ABC$, D and E are points on AB and AC respectively such that $DE \parallel BC$. If AD = 4 cm, DB = 6 cm and the area of $\triangle ABC$ = 75 cm², find (a) the ratio AD : AB, (b) the ratio of the areas of $\triangle ADE$ and $\triangle ABC$, and (c) the area of $\triangle ADE$ and the area of the trapezium DBCE. **[6]**

(ii) The 4th term of an A.P. is 11 and the 8th term is 23. Find the first term, the common difference, and the sum of the first 15 terms. **[4]**

Q8. (Use graph paper for this question.) **[10]**

The marks obtained by 80 students in a test are given below.

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70
No. of students	5	8	14	20	18	9	6

Draw an ogive (cumulative frequency curve) for the data and use it to estimate: (i) the median, (ii) the lower quartile (Q_1), and (iii) the number of students who scored more than 60 marks.

Q9. **[10]**

(i) (Use graph paper.) Draw a histogram for the following distribution and hence estimate the mode.

Class	0–10	10–20	20–30	30–40	40–50	50–60
Frequency	4	9	16	10	7	4

(ii) If $A = \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix}$, find A^2 and verify that $A^2 - 5A + 6I = 0$, where I is the 2 × 2 identity matrix. **[4] [6]**

SOLUTIONS & MARKING SCHEME — SET 1

Q1. MCQ answers

(i) **(c) ₹240** — 12% of 2000. (ii) **(d) P·n(n+1)/2**. (iii) **(b) {1,2,3,4,5}** — $2x \le 10 \Rightarrow x \le 5$, $x \in N$. (iv) **(b) real and equal** — $D = (-4)^2 - 4 \cdot 1 \cdot 4 = 0$. (v) **(a) 6** — $4 - 10 + k = 0$. (vi) **(c) 12** — $\sqrt{(9 \times 16)}$. (vii) **(b) 1** — $p(1) = 1 - 2 + 3 - 1$. (viii) **(a) 2** — $1 - 3 + k = 0$. (ix) **(b) 2 × 2**. (x) **(c) 2** — 2×1. (xi) **(c) 29** — $2 + 9(3)$. (xii) **(b) 3**. (xiii) **(c) 9 : 25**. (xiv) **(c) 2/3** — BPT. (xv) **(b) 10–20** — highest frequency 12.

Q2 (i) — GST

Selling price = 15000 × (1 + 20/100) = **₹18,000**. GST = 12% of 18000 = ₹2,160.

As it is an intra-state sale: CGST = SGST = 6% of 18000 = **₹1,080 each**.

Total amount paid by consumer = 18000 + 2160 = **₹20,160**.

Q2 (ii) — Recurring Deposit

Interest $I = P \times \frac{n(n+1)}{2} \times \frac{r}{12 \times 100} = 800 \times \frac{24 \times 25}{2} \times \frac{10}{1200} = 800 \times 300 \times \frac{10}{1200} =$ **₹2,000**.

Maturity value = P·n + I = 800 × 24 + 2000 = 19200 + 2000 = **₹21,200**.

Q2 (iii) — Matrix product

$$AB = \begin{vmatrix} 1\cdot2+3\cdot1 & 1\cdot(-2)+3\cdot4 \\ 2\cdot2+(-1)\cdot1 & 2\cdot(-2)+(-1)\cdot4 \end{vmatrix} = \begin{vmatrix} 5 & 10 \\ 3 & -8 \end{vmatrix}, \quad \therefore AB = \begin{vmatrix} \mathbf{5} & \mathbf{10} \\ \mathbf{3} & \mathbf{-8} \end{vmatrix}.$$

Q3 (i) — Inequation

$-8 < 3x - 2 \le 7 \Rightarrow$ add 2: $-6 < 3x \le 9 \Rightarrow$ divide by 3: **$-2 < x \le 3$**. Solution set = $\{x : -2 < x \le 3, x \in R\}$ = **(−2, 3]**.

Number line: open circle at −2, closed dot at 3, thick line between.

Q3 (ii) — A.P.

Here $a = 5$, $d = 4$. $t_n = a + (n-1)d = 81 \Rightarrow 5 + (n-1)4 = 81 \Rightarrow (n-1)4 = 76 \Rightarrow n - 1 = 19 \Rightarrow \mathbf{n = 20}$ (the 20th term).

$S_{15} = \frac{15}{2}[2a + (15-1)d] = \frac{15}{2}[10 + 14\times4] = \frac{15}{2}(66) =$ **495**.

Q3 (iii) — Quadratic (word problem)

Let the numbers be n and $(n+1)$. Then $n^2 + (n+1)^2 = 313 \Rightarrow 2n^2 + 2n + 1 = 313 \Rightarrow 2n^2 + 2n - 312 = 0 \Rightarrow \mathbf{n^2 + n - 156 = 0}$.

$(n + 13)(n - 12) = 0 \Rightarrow n = 12$ (reject $n = -13$, not natural). $\therefore$ the numbers are **12 and 13**.

Q4 (i) — Quadratic formula

$3x^2 - 5x - 4 = 0$, $a=3$, $b=-5$, $c=-4$. $D = 25 + 48 = 73$, $\sqrt{73} \approx 8.544$.

$x = \frac{5 \pm \sqrt{73}}{6} = \frac{5 \pm 8.544}{6} \Rightarrow x = \frac{13.544}{6}$ or $\frac{-3.544}{6} \Rightarrow$ **$x \approx 2.26$ or $x \approx -0.59$**.

Q4 (ii) — Proportion

Let the number be x. Then $(6+x)$, $(15+x)$, $(20+x)$, $(43+x)$ are in proportion $\Rightarrow \frac{6+x}{15+x} = \frac{20+x}{43+x}$.

$(6+x)(43+x) = (15+x)(20+x) \Rightarrow 258 + 49x + x^2 = 300 + 35x + x^2 \Rightarrow 14x = 42 \Rightarrow$ **$x = 3$**.

Q4 (iii) — Properties of proportion (Componendo & Dividendo)

$\frac{x^3 + 3x}{3x^2 + 1} = \frac{341}{91}$. Applying componendo and dividendo:

$\frac{(x^3 + 3x) + (3x^2 + 1)}{(x^3 + 3x) - (3x^2 + 1)} = \frac{341 + 91}{341 - 91} \Rightarrow \frac{x^3 + 3x^2 + 3x + 1}{x^3 - 3x^2 + 3x - 1} = \frac{432}{250} \Rightarrow \frac{(x+1)^3}{(x-1)^3} = \frac{216}{125}$.

Taking cube roots: $\frac{x+1}{x-1} = \frac{6}{5} \Rightarrow 5(x + 1) = 6(x - 1) \Rightarrow 5x + 5 = 6x - 6 \Rightarrow$ **$x = 11$**.

Q5 (i) — Remainder theorem

$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$. Remainder $= p(\frac{1}{2}) = 2\cdot\frac{1}{8} + 3\cdot\frac{1}{4} - 5\cdot\frac{1}{2} + 7 = \frac{1}{4} + \frac{3}{4} - \frac{5}{2} + 7 =$ **$\frac{11}{2} = 5.5$**.

Q5 (ii) — Factor theorem

$p(x) = x^3 - 7x + 6$. $p(1) = 1 - 7 + 6 = 0 \Rightarrow (x - 1)$ is a factor.

Dividing: $x^3 - 7x + 6 = (x - 1)(x^2 + x - 6) = (x - 1)(x + 3)(x - 2)$. $\therefore$ **$(x - 1)(x - 2)(x + 3)$**.

Q5 (iii) — Matrices

$2B = \begin{vmatrix} 2 & 0 \\ 4 & -2 \end{vmatrix}$. $A - 2B = \begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix} - \begin{vmatrix} 2 & 0 \\ 4 & -2 \end{vmatrix} = \begin{vmatrix} 0 & 3 \\ -3 & -2 \end{vmatrix} = \begin{vmatrix} \mathbf{0} & \mathbf{3} \\ \mathbf{-3} & \mathbf{-2} \end{vmatrix}$.

Q6 (i) — GST (shopkeeper)

Cost price (net of discount) = 8000 × (1 − 25/100) = ₹6,000. GST paid on purchase = 18% of 6000 = ₹1,080.

(a) Price paid by shopkeeper inclusive of GST = 6000 + 1080 = **₹7,080**.

GST charged on sale = 18% of 8000 = ₹1,440. Net GST to government = 1440 − 1080 = ₹360.

(b) CGST to Central Government = $\frac{360}{2}$ = **₹180** (SGST ₹180 to State).

(c) Amount paid by the consumer = 8000 + 1440 = **₹9,440**.

Q6 (ii) — Recurring Deposit (find rate)

Deposits = 2500 × 12 = ₹30,000. Interest = MV − deposits = 31625 − 30000 = ₹1,625.

$I = P \times \frac{n(n+1)}{2} \times \frac{r}{1200} \Rightarrow 1625 = 2500 \times \frac{12\times13}{2} \times \frac{r}{1200} = 2500 \times 78 \times \frac{r}{1200} = 162.5\,r$.

$r = \frac{1625}{162.5}$ = **10% per annum**.

Q7 (i) — Similarity

AD = 4, DB = 6 ⇒ AB = 10. (a) AD : AB = 4 : 10 = **2 : 5**.

Since DE ∥ BC, △ADE ~ △ABC. (b) $\frac{ar(\triangle ADE)}{ar(\triangle ABC)} = (AD/AB)^2 = (2/5)^2$ = **4 : 25**.

(c) $ar(\triangle ADE) = \frac{4}{25} \times 75$ = **12 cm²**. ar(trapezium DBCE) = 75 − 12 = **63 cm²**.

Q7 (ii) — A.P.

$a + 3d = 11$ …(1), $a + 7d = 23$ …(2). (2) − (1): $4d = 12 \Rightarrow d = 3$; then $a = 11 - 9 = 2$.

A.P.: 2, 5, 8, … $S_{15} = \frac{15}{2}[2(2) + 14(3)] = \frac{15}{2}(4 + 42) = \frac{15}{2}(46)$ = **345**.

Q8 — Ogive (N = 80)

Cumulative frequencies (plot upper class-boundary vs c.f.):

≤	10	20	30	40	50	60	70
c.f.	5	13	27	47	65	74	80

(i) Median = value at $\frac{N}{2}$ = 40th student. From the ogive (or by interpolation, median class 30–40): median $= 30 + \frac{40-27}{20}\times 10 = 30 + 6.5 =$ **36.5 marks**.

(ii) Lower quartile Q_1 = value at $\frac{N}{4}$ = 20th student (class 20–30): $Q_1 = 20 + \frac{20-13}{14}\times 10 = 20 + 5 =$ **25 marks**.

(iii) Students scoring more than 60 = 80 − (c.f. up to 60) = 80 − 74 = **6 students**.

Q9 (i) — Histogram & mode

Modal class = 20–30 (highest frequency 16). Draw the histogram; join the top-left corner of the modal bar to the top-left corner of the next bar, and the top-right corner of the modal bar to the top-right corner of the previous bar; the x-value under their intersection is the mode.

By formula (as a check): mode $= L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2}\times h = 20 + \frac{16-9}{32-9-10}\times 10 = 20 + \frac{70}{13} \approx$ **25.4 (≈ 25.38)**.

Q9 (ii) — Matrices identity

$$A^2 = \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix}\begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = \begin{vmatrix} 4 & 2+3 \\ 0 & 9 \end{vmatrix} = \begin{vmatrix} 4 & 5 \\ 0 & 9 \end{vmatrix}.$$

$$SA = \begin{vmatrix} 10 & 5 \\ 0 & 15 \end{vmatrix},\quad 6I = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix},\quad A^2 - SA + 6I = \begin{vmatrix} 4-10+6 & 5-5+0 \\ 0 & 9-15+6 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} =$$ **0**. ✓ Verified.