KUMAR CLASSES

NCERT SOLUTIONS · CLASS 12 MATHEMATICS

Exercise 7.9

Chapter 7 — Integrals · Definite Integrals by Substitution

Q1–8: evaluate by substitution (change the limits with the variable). Q9–10: choose the correct option.

1. $\int_0^1 \frac{x}{x^2+1}\,dx$

Put $t = x^2+1$ ($dt = 2x\,dx$); limits t: $1 \to 2$.

Sol. $= \frac{1}{2}\left[\log(x^2+1)\right]_0^1 = \frac{1}{2}\log 2$

2. $\int_0^{\pi/2} \sqrt{\sin\varphi}\,\cos^5\varphi\,d\varphi$

$\cos^5\varphi = (1-\sin^2\varphi)^2\cos\varphi$; put $t = \sin\varphi$, limits t: $0 \to 1$.

Sol. $= \int_0^1 (t^{1/2} - 2t^{5/2} + t^{9/2})\,dt = \frac{2}{3} - \frac{4}{7} + \frac{2}{11} = \frac{64}{231}$

3. $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

$\sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x$ on $[0,1]$; integrate by parts.

Sol. $= 2\left[x\tan^{-1}x - \frac{1}{2}\log(1+x^2)\right]_0^1 = \frac{\pi}{2} - \log 2$

4. $\int_0^2 x\sqrt{x+2}\,dx$

Put $x+2 = t^2$ (given), so $x = t^2-2$, $dx = 2t\,dt$; limits t: $\sqrt{2} \to 2$.

Sol. $= 2\int_{\sqrt{2}}^{2}(t^4 - 2t^2)\,dt = \frac{32 + 16\sqrt{2}}{15}$

5. $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$

Put $t = \cos x$ ($dt = -\sin x\,dx$); limits t: $1 \to 0$.

Sol. $= \int_0^1 \frac{dt}{1+t^2} = \left[\tan^{-1}t\right]_0^1 = \frac{\pi}{4}$

6. $\int_0^2 \dfrac{dx}{x+4-x^2}$

Complete the square: $4+x-x^2 = \frac{17}{4} - (x-\frac{1}{2})^2$.

Sol. $= \dfrac{1}{\sqrt{17}} \left[\log\left| \dfrac{(\sqrt{17}+2x-1)}{(\sqrt{17}-2x+1)} \right| \right]_0^2 = \dfrac{1}{\sqrt{17}} \log \dfrac{5+\sqrt{17}}{5-\sqrt{17}}$

7. $\int_{-1}^1 \dfrac{dx}{x^2+2x+5}$

$x^2+2x+5 = (x+1)^2+4$; standard $\tan^{-1}$ form.

Sol. $= \dfrac{1}{2} \left[\tan^{-1} \dfrac{x+1}{2} \right]_{-1}^1 = \dfrac{1}{2} \cdot \dfrac{\pi}{4} = \dfrac{\pi}{8}$

8. $\int_1^2 \left(\dfrac{1}{x} - \dfrac{1}{2x^2} \right) e^{2x}\, dx$

Notice $\dfrac{d}{dx} \left(\dfrac{e^{2x}}{2x} \right) = \left(\dfrac{1}{x} - \dfrac{1}{2x^2} \right) e^{2x}$.

Sol. $= \left[\dfrac{e^{2x}}{2x} \right]_1^2 = \dfrac{e^4}{4} - \dfrac{e^2}{2}$

9. The value of $\int_{1/3}^1 \dfrac{(x-x^3)^{1/3}}{x^4}\, dx$ is

Take x^3 out: $(x-x^3)^{1/3} = x\left(\dfrac{1}{x^2} - 1 \right)^{1/3}$; put $t = \dfrac{1}{x^2} - 1$ (limits t: $8 \to 0$).

Sol. $= \dfrac{1}{2} \int_0^8 t^{1/3}\, dt = \dfrac{3}{8} \cdot 8^{4/3} = \dfrac{3}{8} \cdot 16 = 6$

$\therefore$ Option (A)

10. If $f(x) = \int_0^x t \sin t\, dt$, then $f'(x)$ is

By the Fundamental Theorem of Calculus, $f'(x)$ is the integrand with the upper limit substituted.

Sol. $f'(x) = x \sin x$

$\therefore$ Option (B)