KUMAR CLASSES

NCERT SOLUTIONS · CLASS 12 MATHEMATICS

Exercise 7.8

Chapter 7 — Integrals · Evaluation of Definite Integrals

Q1–20: evaluate using the Fundamental Theorem of Calculus (find the antiderivative, then apply the limits). Q21–22: choose the correct option.

1. $\int_{-1}^{1} (x+1)\, dx$

Antiderivative $\frac{x^2}{2} + x$, evaluated at the limits.

Sol. $= \left[\frac{x^2}{2} + x \right]_{-1}^{1} = \frac{3}{2} - \left(-\frac{1}{2}\right) = \mathbf{2}$

---

2. $\int_{2}^{3} \frac{1}{x}\, dx$

$\int \frac{1}{x}\, dx = \log x$.

Sol. $= [\log x]_{2}^{3} = \log \frac{3}{2}$

---

3. $\int_{1}^{2} (4x^3 - 5x^2 + 6x + 9)\, dx$

Integrate term by term, then apply limits.

Sol. $= \left[x^4 - \frac{5}{3}x^3 + 3x^2 + 9x \right]_{1}^{2} = \frac{64}{3}$

---

4. $\int_{0}^{\pi/4} \sin 2x\, dx$

$\int \sin 2x\, dx = -\frac{1}{2}\cos 2x$.

Sol. $= \left[-\frac{1}{2}\cos 2x \right]_{0}^{\pi/4} = 0 + \frac{1}{2} = \frac{1}{2}$

---

5. $\int_{0}^{\pi/2} \cos 2x\, dx$

$\int \cos 2x\, dx = \frac{1}{2}\sin 2x$.

Sol. $= \left[\frac{1}{2}\sin 2x \right]_{0}^{\pi/2} = \mathbf{0}$

---

6. $\int_4^5 e^x\,dx$

$\int e^x dx = e^x.$

Sol. $= [\,e^x\,]_4^5 = e^5 - e^4$

7. $\int_0^{\pi/4} \tan x\,dx$

$\int \tan x\,dx = \log|\sec x|.$

Sol. $= [\,\log|\sec x|\,]_0^{\pi/4} = \log\sqrt{2} = \dfrac{1}{2}\log 2$

8. $\int_{\pi/6}^{\pi/4} \operatorname{cosec} x\,dx$

$\int \operatorname{cosec} x\,dx = \log\left|\tan\dfrac{x}{2}\right|.$

Sol. $= \left[\,\log\left|\tan\dfrac{x}{2}\right|\,\right]_{\pi/6}^{\pi/4} = \log\dfrac{\sqrt{2}-1}{2-\sqrt{3}}$

9. $\int_0^1 \dfrac{1}{\sqrt{1-x^2}}\,dx$

$\int \dfrac{dx}{\sqrt{1-x^2}} = \sin^{-1}x.$

Sol. $= [\,\sin^{-1}x\,]_0^1 = \dfrac{\pi}{2}$

10. $\int_0^1 \dfrac{1}{1+x^2}\,dx$

$\int \dfrac{dx}{1+x^2} = \tan^{-1}x.$

Sol. $= [\,\tan^{-1}x\,]_0^1 = \dfrac{\pi}{4}$

11. $\int_2^3 \dfrac{1}{x^2-1}\,dx$

$\int \dfrac{dx}{x^2-1} = \dfrac{1}{2}\log\left|\dfrac{x-1}{x+1}\right|.$

Sol. $= \left[\,\dfrac{1}{2}\log\left|\dfrac{x-1}{x+1}\right|\,\right]_2^3 = \dfrac{1}{2}\log\dfrac{3}{2}$

12. $\int_0^{\pi/2} \cos^2 x\,dx$

$\cos^2 x = \dfrac{1+\cos 2x}{2}.$

Sol. $= \dfrac{1}{2}\left[\,x + \dfrac{\sin 2x}{2}\,\right]_0^{\pi/2} = \dfrac{\pi}{4}$

13. $\int_2^3 \frac{x}{x^2+1}\,dx$

Put $t = x^2+1$; integral $= \frac{1}{2}\log(x^2+1)$.

Sol. $= \left[\frac{1}{2}\log(x^2+1)\right]_2^3 = \frac{1}{2}(\log 10 - \log 5) = \frac{1}{2}\log 2$

---

14. $\int_0^1 \frac{2x+3}{5x^2+1}\,dx$

Split into $\frac{2x}{5x^2+1}$ (log part) and $\frac{3}{5x^2+1}$ ($\tan^{-1}$ part).

Sol. $= \left[\frac{1}{5}\log(5x^2+1) + \frac{3}{\sqrt{5}}\tan^{-1}(\sqrt{5}x)\right]_0^1$

$= \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$

---

15. $\int_0^1 x\, e^{x^2}\,dx$

Put $t = x^2$; integral $= \frac{1}{2}e^{x^2}$.

Sol. $= \left[\frac{1}{2}e^{x^2}\right]_0^1 = \frac{1}{2}(e-1)$

---

16. $\int_1^2 \frac{5x^2}{x^2+4x+3}\,dx$

Improper → $5x^2 = 5(x^2+4x+3) - (20x+15)$; then partial fractions.

Sol. $= \left[5x + \frac{5}{2}\log(x+1) - \frac{45}{2}\log(x+3)\right]_1^2$

$= 5 + \frac{5}{2}\log\frac{3}{2} - \frac{45}{2}\log\frac{5}{4}$

---

17. $\int_0^{\pi/4} (2\sec^2 x + x^3 + 2)\,dx$

Integrate term by term.

Sol. $= \left[2\tan x + \frac{x^4}{4} + 2x\right]_0^{\pi/4} = 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}$

---

18. $\int_0^{\pi} \left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right)dx$

$\sin^2\frac{x}{2} - \cos^2\frac{x}{2} = -\cos x$.

Sol. $= -\left[\sin x\right]_0^{\pi} = 0$

---

19. $\int_0^2 \frac{6x+3}{x^2+4}\,dx$

Split into $3\cdot\frac{2x}{x^2+4}$ (log) and $\frac{3}{x^2+4}$ ($\tan^{-1}$).

Sol. $= \left[\, 3\log(x^2+4) + \frac{3}{2}\tan^{-1}\frac{x}{2} \,\right]_0^2 = \mathbf{3\log 2 + \frac{3\pi}{8}}$

20. $\int_0^1 \left(x e^x + \sin\frac{\pi x}{4} \right) dx$

$\int x e^x dx = (x-1)e^x$; $\int \sin\frac{\pi x}{4}\,dx = -\frac{4}{\pi}\cos\frac{\pi x}{4}$.

Sol. $= \left[\, (x-1)e^x - \frac{4}{\pi}\cos\frac{\pi x}{4} \,\right]_0^1 = \mathbf{1 + \frac{4-2\sqrt{2}}{\pi}}$

21. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

$= [\tan^{-1}x] = \tan^{-1}\sqrt{3} - \tan^{-1}1 = \frac{\pi}{3} - \frac{\pi}{4}$.

Sol. $= \mathbf{\frac{\pi}{12}}$

$\therefore$ Option (D)

22. $\int_0^{2/3} \frac{dx}{4+9x^2}$ equals

$= \frac{1}{6}\tan^{-1}\frac{3x}{2}$; at $x = \frac{2}{3}$ gives $\frac{1}{6}\cdot\frac{\pi}{4}$.

Sol. $= \mathbf{\frac{\pi}{24}}$

$\therefore$ Option (C)