



KUMAR CLASSES

NCERT SOLUTIONS · CLASS 12 MATHEMATICS

Exercise 7.7

Chapter 7 — Integrals · Integrals of the Type $\sqrt{(x^2 \pm a^2)}$, $\sqrt{(a^2 - x^2)}$ Q1-9: complete the square, then apply the standard $\sqrt{}$ formulae. Q10-11: choose the correct option.

1. $\int \sqrt{4-x^2} \, dx$

Form $\int \sqrt{a^2 - x^2}$ with $a = 2$.

Sol. $= \frac{x}{2} \sqrt{4-x^2} + 2 \sin^{-1} \frac{x}{2} + C$

2. $\int \sqrt{1-4x^2} \, dx$

Write as $\sqrt{1-(2x)^2}$; use $\sqrt{a^2 - u^2}$ with $u = 2x$.

Sol. $= \frac{x}{2} \sqrt{1-4x^2} + \frac{1}{4} \sin^{-1}(2x) + C$

3. $\int \sqrt{x^2+4x+6} \, dx$

Complete the square: $x^2+4x+6 = (x+2)^2+2 \rightarrow \sqrt{x^2+a^2}$ form.

Sol. $= \frac{x+2}{2} \sqrt{x^2+4x+6} + \log | (x+2) + \sqrt{x^2+4x+6} | + C$

4. $\int \sqrt{x^2+4x+1} \, dx$

 $x^2+4x+1 = (x+2)^2 - 3 \rightarrow \sqrt{x^2-a^2}$ form.

Sol. $= \frac{x+2}{2} \sqrt{x^2+4x+1} - \frac{3}{2} \log | (x+2) + \sqrt{x^2+4x+1} | + C$

5. $\int \sqrt{1-4x-x^2} \, dx$

 $1-4x-x^2 = 5 - (x+2)^2 \rightarrow \sqrt{a^2 - x^2}$ form.

Sol. $= \frac{x+2}{2} \sqrt{1-4x-x^2} + \frac{5}{2} \sin^{-1} \frac{x+2}{\sqrt{5}} + C$

6. $\int \sqrt{x^2+4x-5} \, dx$

 $x^2+4x-5 = (x+2)^2 - 9$.

Sol. $= \frac{x+2}{2} \sqrt{x^2+4x-5} - \frac{9}{2} \log | (x+2) + \sqrt{x^2+4x-5} | + C$

7. $\int \sqrt{1+3x-x^2} \, dx$

$$1+3x-x^2 = \frac{13}{4} - \left(x - \frac{3}{2}\right)^2.$$

Sol. $= \frac{2x-3}{4} \sqrt{1+3x-x^2} + \frac{13}{8} \sin^{-1} \frac{2x-3}{\sqrt{13}} + C$

8. $\int \sqrt{x^2+3x} \, dx$

$$x^2+3x = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4}.$$

Sol. $= \frac{2x+3}{4} \sqrt{x^2+3x} - \frac{9}{8} \log \left| \left(x + \frac{3}{2}\right) + \sqrt{x^2+3x} \right| + C$

9. $\int \sqrt{1+\frac{x^2}{9}} \, dx$

$$= \frac{1}{3} \sqrt{9+x^2} \rightarrow \sqrt{x^2+a^2} \text{ form with } a = 3.$$

Sol. $= \frac{x}{2} \sqrt{1+\frac{x^2}{9}} + \frac{3}{2} \log |x + \sqrt{9+x^2}| + C$

10. $\int \sqrt{1+x^2} \, dx$ equals

Standard result $\int \sqrt{x^2+a^2} \, dx$ with $a = 1$.

Sol. $= \frac{x}{2} \sqrt{1+x^2} + \frac{1}{2} \log(x + \sqrt{1+x^2}) + C$

$\therefore$ Option (A)

11. $\int \sqrt{x^2-8x+7} \, dx$ equals

$x^2-8x+7 = (x-4)^2 - 9$; apply the $\sqrt{x^2-a^2}$ formula.

Sol. $= \frac{1}{2} (x-4) \sqrt{x^2-8x+7} - \frac{9}{2} \log | (x-4) + \sqrt{x^2-8x+7} | + C$

$\therefore$ Option (D)