KUMAR CLASSES

NCERT SOLUTIONS · CLASS 12 MATHEMATICS

Exercise 7.6

Chapter 7 — Integrals · Integration by Parts

Q1–22: integrate by parts (Q16–20 use the $\int e^x[f(x)+f'(x)]dx = e^x f(x)$ rule). Q23–24: choose the correct option.

1. $\int x \sin x \, dx$

By parts with $u = x$, $dv = \sin x\, dx$ (so $v = -\cos x$).

Sol. $= -x \cos x + \sin x + C$

2. $\int x \sin 3x \, dx$

By parts with $u = x$, $dv = \sin 3x\, dx$.

Sol. $= -\dfrac{x}{3} \cos 3x + \dfrac{1}{9} \sin 3x + C$

3. $\int x^2 e^x \, dx$

By parts twice ($u = x^2$, then $u = x$).

Sol. $= e^x(x^2 - 2x + 2) + C$

4. $\int x \log x \, dx$

By parts with $u = \log x$, $dv = x\, dx$.

Sol. $= \dfrac{x^2}{2} \log x - \dfrac{x^2}{4} + C$

5. $\int x \log 2x \, dx$

By parts with $u = \log 2x$, $dv = x\, dx$ ($\dfrac{d}{dx} \log 2x = \dfrac{1}{x}$).

Sol. $= \dfrac{x^2}{2} \log 2x - \dfrac{x^2}{4} + C$

6. $\int x^2 \log x \, dx$

By parts with $u = \log x$, $dv = x^2\, dx$.

Sol. $= \dfrac{x^3}{3} \log x - \dfrac{x^3}{9} + C$

7. $\int x \sin^{-1}x \, dx$

By parts: $u = \sin^{-1}x$, $dv = x\,dx$; then evaluate $\int \frac{x^2}{\sqrt{1-x^2}}$.

Sol. $= \dfrac{2x^2-1}{4} \sin^{-1}x + \dfrac{x\sqrt{1-x^2}}{4} + C$

8. $\int x \tan^{-1}x \, dx$

By parts: $u = \tan^{-1}x$, $dv = x\,dx$.

Sol. $= \dfrac{x^2+1}{2} \tan^{-1}x - \dfrac{x}{2} + C$

9. $\int x \cos^{-1}x \, dx$

By parts: $u = \cos^{-1}x$, $dv = x\,dx$.

Sol. $= \dfrac{2x^2-1}{4} \cos^{-1}x - \dfrac{x\sqrt{1-x^2}}{4} + C$

10. $\int (\sin^{-1}x)^2 \, dx$

Take 1 as the second function; by parts twice.

Sol. $= x(\sin^{-1}x)^2 + 2\sqrt{1-x^2}\,\sin^{-1}x - 2x + C$

11. $\int \dfrac{x \cos^{-1}x}{\sqrt{1-x^2}} \, dx$

By parts: $u = \cos^{-1}x$, $dv = \frac{x}{\sqrt{1-x^2}}dx$ ($v = -\sqrt{1-x^2}$).

Sol. $= -\sqrt{1-x^2}\,\cos^{-1}x - x + C$

12. $\int x \sec^2x \, dx$

By parts: $u = x$, $dv = \sec^2x\,dx$ ($v = \tan x$).

Sol. $= x \tan x + \log|\cos x| + C$

13. $\int \tan^{-1}x \, dx$

By parts with 1 as the second function: $u = \tan^{-1}x$, $dv = dx$.

Sol. $= x \tan^{-1}x - \dfrac{1}{2} \log(1+x^2) + C$

14. $\int x(\log x)^2\,dx$

By parts: $u = (\log x)^2$, $dv = x\,dx$; reduces to $\int x \log x\,dx$.

Sol. $= \dfrac{x^2}{2}(\log x)^2 - \dfrac{x^2}{2}\log x + \dfrac{x^2}{4} + C$

15. $\int (x^2+1)\log x\,dx$

Split into $\int x^2 \log x\,dx + \int \log x\,dx$, each by parts.

Sol. $= \dfrac{x^3}{3}\log x + x\log x - \dfrac{x^3}{9} - x + C$

16. $\int e^x(\sin x + \cos x)\,dx$

Recognise $e^x(\sin x + \cos x) = \dfrac{d}{dx}(e^x \sin x)$.

Sol. $= e^x \sin x + C$

17. $\int \dfrac{x\,e^x}{(1+x)^2}\,dx$

Type $\int e^x[f + f']$: $\dfrac{x}{(1+x)^2} = \dfrac{1}{1+x} - \dfrac{1}{(1+x)^2}$, $f = \dfrac{1}{1+x}$.

Sol. $= \dfrac{e^x}{1+x} + C$

18. $\int e^x \dfrac{1+\sin x}{1+\cos x}\,dx$

Type $\int e^x[f + f']$: $\dfrac{1+\sin x}{1+\cos x} = \tan\dfrac{x}{2} + \dfrac{1}{2}\sec^2\dfrac{x}{2}$.

Sol. $= e^x \tan\dfrac{x}{2} + C$

19. $\int e^x\left(\dfrac{1}{x} - \dfrac{1}{x^2}\right)dx$

Type $\int e^x[f + f']$ with $f = \dfrac{1}{x}$ $\left(f' = -\dfrac{1}{x^2}\right)$.

Sol. $= \dfrac{e^x}{x} + C$

20. $\int \dfrac{(x-3)e^x}{(x-1)^3}\,dx$

Type $\int e^x[f + f']$: $\dfrac{x-3}{(x-1)^3} = \dfrac{1}{(x-1)^2} - \dfrac{2}{(x-1)^3}$, $f = \dfrac{1}{(x-1)^2}$.

Sol. $= \dfrac{e^x}{(x-1)^2} + C$

21. $\int e^{2x} \sin x \, dx$

By parts twice and solve for the integral ($e^{ax}\sin bx$ type).

Sol. $= \dfrac{e^{2x}}{5}(2 \sin x - \cos x) + C$

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22. $\int \sin^{-1}\left(\dfrac{2x}{1+x^2} \right) dx$

Since $\sin^{-1}\dfrac{2x}{1+x^2} = 2 \tan^{-1}x$, integrate $2 \tan^{-1}x$ by parts.

Sol. $= 2x \tan^{-1}x - \log(1+x^2) + C$

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23. $\int x^2 e^{x^3} dx$ equals

Put $t = x^3$, so $dt = 3x^2 dx$ (substitution, not by parts).

Sol. $= \dfrac{1}{3} e^{x^3} + C$

∴ Option (A)

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24. $\int e^x \sec x(1 + \tan x) \, dx$ equals

$e^x(\sec x + \sec x \tan x)$ is of the form $e^x[f + f']$ with $f = \sec x$.

Sol. $= e^x \sec x + C$

∴ Option (B)