KUMAR CLASSES

NCERT SOLUTIONS · CLASS 12 MATHEMATICS

Exercise 7.5

Chapter 7 — Integrals · Integration by Partial Fractions

Q1–21: integrate the rational functions (split into partial fractions; divide first if improper). Q22–23: choose the correct option.

1. $\int \frac{x}{(x+1)(x+2)}\,dx$

Partial fractions: $\frac{x}{(x+1)(x+2)} = -\frac{1}{x+1} + \frac{2}{x+2}$.

Sol. $= 2\log|x+2| - \log|x+1| + C$

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2. $\int \frac{1}{x^2-9}\,dx$

$= \frac{1}{(x-3)(x+3)}$; *standard* $\frac{1}{x^2-a^2}$ *form.*

Sol. $= \frac{1}{6}\log\left|\frac{x-3}{x+3}\right| + C$

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3. $\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$

PF gives $\frac{1}{x-1} - \frac{5}{x-2} + \frac{4}{x-3}$.

Sol. $= \log|x-1| - 5\log|x-2| + 4\log|x-3| + C$

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4. $\int \frac{x}{(x-1)(x-2)(x-3)}\,dx$

PF: $\frac{1/2}{x-1} - \frac{2}{x-2} + \frac{3/2}{x-3}$.

Sol. $= \frac{1}{2}\log|x-1| - 2\log|x-2| + \frac{3}{2}\log|x-3| + C$

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5. $\int \frac{2x}{x^2+3x+2}\,dx$

$= \frac{2x}{(x+1)(x+2)}$; *PF:* $-\frac{2}{x+1} + \frac{4}{x+2}$.

Sol. $= -2\log|x+1| + 4\log|x+2| + C$

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6. $\int \frac{1-x^2}{x(1-2x)}\,dx$

Improper → divide first: $= \frac{1}{2} + \frac{1}{x} + \frac{3/2}{1-2x}$.

Sol. $= \frac{x}{2} + \log|x| - \frac{3}{4}\log|1-2x| + C$

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7. $\int \frac{x}{(x^2+1)(x-1)}\,dx$

PF: $\frac{-\frac{1}{2}x+\frac{1}{2}}{x^2+1} + \frac{\frac{1}{2}}{x-1}$.

Sol. $= \frac{1}{2}\log|x-1| - \frac{1}{4}\log(x^2+1) + \frac{1}{2}\tan^{-1}x + C$

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8. $\int \frac{x}{(x-1)^2(x+2)}\,dx$

Repeated factor PF: $\frac{2/9}{x-1} + \frac{1/3}{(x-1)^2} - \frac{2/9}{x+2}$.

Sol. $= \frac{2}{9}\log|x-1| - \frac{1}{3(x-1)} - \frac{2}{9}\log|x+2| + C$

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9. $\int \frac{3x+5}{x^3-x^2-x+1}\,dx$

Factor: $x^3-x^2-x+1 = (x-1)^2(x+1)$; *then PF.*

Sol. $= -\frac{1}{2}\log|x-1| - \frac{4}{x-1} + \frac{1}{2}\log|x+1| + C$

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10. $\int \frac{2x-3}{(x^2-1)(2x+3)}\,dx$

Factor $x^2-1 = (x-1)(x+1)$; *PF over three factors.*

Sol. $= -\frac{1}{10}\log|x-1| + \frac{5}{2}\log|x+1| - \frac{12}{5}\log|2x+3| + C$

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11. $\int \frac{5x}{(x+1)(x^2-4)}\,dx$

$= \frac{5x}{(x+1)(x-2)(x+2)}$; *PF.*

Sol. $= \frac{5}{3}\log|x+1| + \frac{5}{6}\log|x-2| - \frac{5}{2}\log|x+2| + C$

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12. $\int \frac{x^3+x+1}{x^2-1}\,dx$

Improper → divide: $= x + \frac{2x+1}{x^2-1}$, *then PF.*

Sol. $= \frac{x^2}{2} + \frac{3}{2}\log|x-1| + \frac{1}{2}\log|x+1| + C$

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13. $\int \frac{2}{(1-x)(1+x^2)}\,dx$

PF: $\frac{1}{1-x} + \frac{x+1}{1+x^2}$.

Sol. $= -\log|1-x| + \frac{1}{2}\log(1+x^2) + \tan^{-1}x + C$

14. $\int \frac{3x-1}{(x+2)^2}\,dx$

Put $u = x+2$: $\frac{3u-7}{u^2} = \frac{3}{u} - \frac{7}{u^2}$.

Sol. $= 3\log|x+2| + \frac{7}{x+2} + C$

15. $\int \frac{1}{x^4-1}\,dx$

Write $\frac{1}{x^4-1} = \frac{1}{2}\left[\frac{1}{x^2-1} - \frac{1}{x^2+1}\right]$.

Sol. $= \frac{1}{4}\log\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\tan^{-1}x + C$

16. $\int \frac{1}{x(x^n+1)}\,dx$

Multiply num & denom by x^{n-1} and put $t = x^n$ ($dt = n\,x^{n-1}dx$).

Sol. $= \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$

17. $\int \frac{\cos x}{(1-\sin x)(2-\sin x)}\,dx$

Put $t = \sin x$ ($dt = \cos x\,dx$), then use PF in t.

Sol. $= \log\left|\frac{2-\sin x}{1-\sin x}\right| + C$

18. $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}\,dx$

Improper → divide: $= 1 + \frac{2}{x^2+3} - \frac{6}{x^2+4}$.

Sol. $= x + \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - 3\tan^{-1}\frac{x}{2} + C$

19. $\int \frac{2x}{(x^2+1)(x^2+3)}\,dx$

Put $t = x^2$ ($dt = 2x\,dx$), then PF in t.

Sol. $= \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + C$

20. $\int \frac{1}{x(x^4-1)}\,dx$

Multiply num & denom by x^3 and put $t = x^4$ ($dt = 4x^3dx$).

Sol. $= \frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right| + C$

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21. $\int \frac{1}{e^x-1}\,dx$

Put $t = e^x$ ($dx = dt/t$), giving $\int \frac{dt}{t(t-1)}$.

Sol. $= \log|e^x-1| - x + C$

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22. $\int \frac{x\,dx}{(x-1)(x-2)}$ **equals**

PF: $\frac{x}{(x-1)(x-2)} = -\frac{1}{x-1} + \frac{2}{x-2}$.

Sol. $= 2\log|x-2| - \log|x-1| = \log\left|\frac{(x-2)^2}{x-1}\right| + C$

∴ Option (B)

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23. $\int \frac{dx}{x(x^2+1)}$ **equals**

PF: $\frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{x^2+1}$.

Sol. $= \log|x| - \frac{1}{2}\log(x^2+1) + C$

∴ Option (A)
