KUMAR CLASSES

NCERT SOLUTIONS · CLASS 12 MATHEMATICS

Exercise 7.4

Chapter 7 — Integrals · Integrals of Some Particular Functions

Q1–23: use the standard forms (complete the square / split the numerator). Q24–25: choose the correct option.

1. $\int \frac{3x^2}{x^6+1}\,dx$

Put $t = x^3$, so $dt = 3x^2\,dx$; reduces to $\int \frac{dt}{t^2+1}$.

Sol. $= \tan^{-1}(x^3) + C$

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2. $\int \frac{1}{\sqrt{1+4x^2}}\,dx$

Form $\int \frac{dx}{\sqrt{1+u^2}}$ with $u = 2x$.

Sol. $= \frac{1}{2}\log\left|\,2x + \sqrt{1+4x^2}\,\right| + C$

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3. $\int \frac{1}{\sqrt{(2-x)^2+1}}\,dx$

Already a perfect square plus 1; use $\int \frac{dX}{\sqrt{X^2+1}}$ with $X = x-2$.

Sol. $= \log\left|\,(x-2) + \sqrt{x^2-4x+5}\,\right| + C$

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4. $\int \frac{1}{\sqrt{9-25x^2}}\,dx$

Form $\int \frac{dx}{\sqrt{a^2-u^2}}$ with $u = 5x$, $a = 3$.

Sol. $= \frac{1}{5}\sin^{-1}\frac{5x}{3} + C$

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5. $\int \frac{3x}{\sqrt{1+2x^4}}\,dx$

Put $t = x^2$ ($dt = 2x\,dx$); reduces to a $\sqrt{t^2}$-type log form.

Sol. $= \frac{3}{2\sqrt{2}}\log\left|\,\sqrt{2}\,x^2 + \sqrt{1+2x^4}\,\right| + C$

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6. $\int \frac{x^2}{\sqrt{1-x^6}}\,dx$

Put $t = x^3$; reduces to $\int \frac{dt}{\sqrt{1-t^2}}$.

Sol. $= \frac{1}{3}\sin^{-1}(x^3) + C$

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7. $\int \frac{x-1}{\sqrt{x^2-1}}\,dx$

Split: $\frac{x}{\sqrt{x^2-1}} - \frac{1}{\sqrt{x^2-1}}$.

Sol. $= \sqrt{x^2-1} - \log\left| x + \sqrt{x^2-1} \right| + C$

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8. $\int \frac{x^2}{\sqrt{x^6+a^6}}\,dx$

Put $t = x^3$; reduces to $\int \frac{dt}{\sqrt{t^2+(a^3)^2}}$.

Sol. $= \frac{1}{3}\log\left| x^3 + \sqrt{x^6+a^6} \right| + C$

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9. $\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}}\,dx$

Put $t = \tan x$, so $dt = \sec^2 x\,dx$.

Sol. $= \log\left| \tan x + \sqrt{\tan^2 x+4} \right| + C$

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10. $\int \frac{1}{\sqrt{x^2+2x+2}}\,dx$

Complete the square: $x^2+2x+2 = (x+1)^2+1$.

Sol. $= \log\left| (x+1) + \sqrt{x^2+2x+2} \right| + C$

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11. $\int \frac{1}{9x^2+6x+5}\,dx$

$9x^2+6x+5 = (3x+1)^2+2^2$; use $\int \frac{du}{u^2+a^2}$.

Sol. $= \frac{1}{6}\tan^{-1}\frac{3x+1}{2} + C$

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12. $\int \frac{1}{\sqrt{7-6x-x^2}}\,dx$

$7-6x-x^2 = 16 - (x+3)^2$; $\sin^{-1}$ form.

Sol. $= \sin^{-1}\frac{x+3}{4} + C$

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13. $\int \frac{1}{\sqrt{(x-1)(x-2)}}\,dx$

$(x-1)(x-2) = (x-\frac{3}{2})^2 - (\frac{1}{2})^2$.

Sol. $= \log\left|\left(x-\frac{3}{2}\right) + \sqrt{x^2-3x+2}\right| + C$

14. $\int \frac{1}{\sqrt{8+3x-x^2}}\,dx$

$8+3x-x^2 = \frac{41}{4} - (x-\frac{3}{2})^2$; $\sin^{-1}$ form.

Sol. $= \sin^{-1}\frac{2x-3}{\sqrt{41}} + C$

15. $\int \frac{1}{\sqrt{(x-a)(x-b)}}\,dx$

$(x-a)(x-b) = (x-\frac{a+b}{2})^2 - (\frac{a-b}{2})^2$.

Sol. $= \log\left|\left(x-\frac{a+b}{2}\right) + \sqrt{(x-a)(x-b)}\right| + C$

16. $\int \frac{4x+1}{\sqrt{2x^2+x-3}}\,dx$

Numerator $4x+1$ is exactly $\frac{d}{dx}(2x^2+x-3)$, so $\int \frac{f'}{\sqrt{f}} = 2\sqrt{f}$.

Sol. $= 2\sqrt{2x^2+x-3} + C$

17. $\int \frac{x+2}{\sqrt{x^2-1}}\,dx$

Write $x+2 = \frac{1}{2}(2x) + 2$, then split.

Sol. $= \sqrt{x^2-1} + 2\log\left|x + \sqrt{x^2-1}\right| + C$

18. $\int \frac{5x-2}{1+2x+3x^2}\,dx$

Write $5x-2 = \frac{5}{6}(6x+2) - \frac{11}{3}$; split into log + $\tan^{-1}$.

Sol. $= \frac{5}{6}\log(3x^2+2x+1) - \frac{11}{3\sqrt{2}}\tan^{-1}\frac{3x+1}{\sqrt{2}} + C$

19. $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}\,dx$

Write $6x+7 = 3(2x-9) + 34$, where $2x-9 = \frac{d}{dx}(x^2-9x+20)$.

Sol. $= 6\sqrt{x^2-9x+20} + 34\log\left|\left(x-\frac{9}{2}\right) + \sqrt{x^2-9x+20}\right| + C$

20. $\int \frac{x+2}{\sqrt{4x-x^2}}\,dx$

Write $x+2 = -\frac{1}{2}(4-2x) + 4$; split into $\sqrt{\ }$ and $\sin^{-1}$ parts.

Sol. $= -\sqrt{4x-x^2} + 4\sin^{-1}\frac{x-2}{2} + C$

21. $\int \frac{x+2}{\sqrt{x^2+2x+3}}\,dx$

Write $x+2 = \frac{1}{2}(2x+2) + 1$; and $x^2+2x+3 = (x+1)^2+2$.

Sol. $= \sqrt{x^2+2x+3} + \log\left|(x+1) + \sqrt{x^2+2x+3}\right| + C$

22. $\int \frac{x+3}{\sqrt{x^2-2x-5}}\,dx$

Write $x+3 = \frac{1}{2}(2x-2) + 4$; and $x^2-2x-5 = (x-1)^2-6$.

Sol. $= \sqrt{x^2-2x-5} + 4\log\left|(x-1) + \sqrt{x^2-2x-5}\right| + C$

23. $\int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$

Write $5x+3 = \frac{5}{2}(2x+4) - 7$; and $x^2+4x+10 = (x+2)^2+6$.

Sol. $= 5\sqrt{x^2+4x+10} - 7\log\left|(x+2) + \sqrt{x^2+4x+10}\right| + C$

24. $\int \frac{dx}{x^2+2x+2}$ equals

$x^2+2x+2 = (x+1)^2+1$, a standard $\tan^{-1}$ form.

Sol. $= \tan^{-1}(x+1) + C$

$\therefore$ Option (B)

25. $\int \frac{dx}{\sqrt{9x-4x^2}}$ equals

$9x-4x^2 = (\frac{9}{8})^2 \cdot 4 - 4(x-\frac{9}{8})^2$; gives a $\sin^{-1}$ form.

Sol. $= \frac{1}{2}\sin^{-1}\frac{8x-9}{9} + C$

$\therefore$ Option (B)