KUMAR CLASSES

NCERT SOLUTIONS · CLASS 12 MATHEMATICS

Exercise 7.3

Chapter 7 — Integrals · Integration using Trigonometric Identities

Q1–22: integrate using suitable trigonometric identities. Q23–24: choose the correct option.

1. $\int \sin^2(2x+5)\,dx$

Use $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$.

Sol. $= \dfrac{1}{2}\int(1-\cos(4x+10))\,dx = \dfrac{x}{2} - \dfrac{\sin(4x+10)}{8} + C$

2. $\int \sin 3x \cos 4x\,dx$

Product-to-sum: $\sin A \cos B = \dfrac{1}{2}[\sin(A+B) + \sin(A-B)]$.

Sol. $= \dfrac{1}{2}\int(\sin 7x - \sin x)\,dx = -\dfrac{\cos 7x}{14} + \dfrac{1}{2}\cos x + C$

3. $\int \cos 2x \cos 4x \cos 6x\,dx$

Apply product-to-sum twice to get $\dfrac{1}{4}[1 + \cos 4x + \cos 8x + \cos 12x]$.

Sol. $= \dfrac{1}{4}\left[x + \dfrac{\sin 4x}{4} + \dfrac{\sin 8x}{8} + \dfrac{\sin 12x}{12}\right] + C$

4. $\int \sin^3(2x+1)\,dx$

Use $\sin^3\theta = \dfrac{3\sin\theta - \sin 3\theta}{4}$, *with* $\theta = 2x+1$.

Sol. $= -\dfrac{3}{8}\cos(2x+1) + \dfrac{1}{24}\cos(6x+3) + C$

5. $\int \sin^3 x \cos^3 x\,dx$

Write $\cos^3 x = (1-\sin^2 x)\cos x$; *put* $t = \sin x$.

Sol. $= \int t^3(1-t^2)\,dt = \dfrac{\sin^4 x}{4} - \dfrac{\sin^6 x}{6} + C$

6. $\int \sin x \sin 2x \sin 3x\,dx$

Combine $\sin x \sin 3x = \dfrac{1}{2}(\cos 2x - \cos 4x)$, *then expand.*

Sol. $= -\dfrac{1}{8}\cos 2x - \dfrac{1}{16}\cos 4x + \dfrac{1}{24}\cos 6x + C$

7. $\int \sin 4x \sin 8x \, dx$

$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$.

Sol. $= \frac{1}{2}\int(\cos 4x - \cos 12x)\, dx = \frac{\sin 4x}{8} - \frac{\sin 12x}{24} + C$

---

8. $\int \frac{1-\cos x}{1+\cos x} \, dx$

Use $1-\cos x = 2\sin^2\frac{x}{2}$ and $1+\cos x = 2\cos^2\frac{x}{2} \Rightarrow \tan^2\frac{x}{2}$.

Sol. $= \int(\sec^2\frac{x}{2} - 1)\, dx = 2\tan\frac{x}{2} - x + C$

---

9. $\int \frac{\cos x}{1+\cos x} \, dx$

Write $\frac{\cos x}{1+\cos x} = 1 - \frac{1}{1+\cos x}$ and $\frac{1}{1+\cos x} = \frac{1}{2}\sec^2\frac{x}{2}$.

Sol. $= x - \tan\frac{x}{2} + C$

---

10. $\int \sin^4 x \, dx$

Use $\sin^4 x = \frac{3 - 4\cos 2x + \cos 4x}{8}$.

Sol. $= \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C$

---

11. $\int \cos^4 2x \, dx$

Use $\cos^4\theta = \frac{3 + 4\cos 2\theta + \cos 4\theta}{8}$ with $\theta = 2x$.

Sol. $= \frac{3x}{8} + \frac{\sin 4x}{8} + \frac{\sin 8x}{64} + C$

---

12. $\int \frac{\sin^2 x}{1+\cos x} \, dx$

$\sin^2 x = (1-\cos x)(1+\cos x)$; cancel $(1+\cos x)$.

Sol. $= \int(1 - \cos x)\, dx = x - \sin x + C$

---

13. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} \, dx$

$\cos 2x - \cos 2\alpha = 2(\cos^2 x - \cos^2\alpha) = 2(\cos x - \cos\alpha)(\cos x + \cos\alpha)$.

Sol. $= 2\int(\cos x + \cos\alpha)\, dx = 2(\sin x + x\cos\alpha) + C$

---

14. $\int \dfrac{\cos x - \sin x}{1+\sin 2x}\,dx$

$1 + \sin 2x = (\sin x + \cos x)^2$; put $t = \sin x + \cos x$.

Sol. $= \int \dfrac{dt}{t^2} = -\dfrac{1}{\sin x + \cos x} + C$

15. $\int \tan^3 2x \sec 2x\, dx$

Write $\tan^3 2x \sec 2x = (\sec^2 2x - 1)\sec 2x \tan 2x$; put $t = \sec 2x$.

Sol. $= \dfrac{1}{2}\int (t^2-1)\,dt = \dfrac{1}{6}\sec^3 2x - \dfrac{1}{2}\sec 2x + C$

16. $\int \tan^4 x\, dx$

$\tan^4 x = \tan^2 x(\sec^2 x - 1)$, then $\tan^2 x = \sec^2 x - 1$ again.

Sol. $= \dfrac{\tan^3 x}{3} - \tan x + x + C$

17. $\int \dfrac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}\,dx$

Split into $\dfrac{\sin x}{\cos^2 x} + \dfrac{\cos x}{\sin^2 x} = \sec x \tan x + \operatorname{cosec} x \cot x$, then integrate.

Sol. $= \int (\sec x \tan x + \operatorname{cosec} x \cot x)\,dx = \sec x - \operatorname{cosec} x + C$

18. $\int \dfrac{\cos 2x + 2\sin^2 x}{\cos^2 x}\,dx$

Since $\cos 2x = 1 - 2\sin^2 x$, the numerator $= 1$, so integrand $= \sec^2 x$.

Sol. $= \int \sec^2 x\,dx = \tan x + C$

19. $\int \dfrac{1}{\sin x \cos^3 x}\,dx$

Multiply numerator by $(\sin^2 x + \cos^2 x) = 1$ and split.

Sol. $= \int \left(\dfrac{\sin x}{\cos^3 x} + \dfrac{1}{\sin x \cos x}\right)dx = \dfrac{1}{2}\sec^2 x + \log|\tan x| + C$

20. $\int \dfrac{\cos 2x}{(\cos x + \sin x)^2}\,dx$

$\cos 2x = (\cos x - \sin x)(\cos x + \sin x)$; cancel one factor.

Sol. $= \int \dfrac{\cos x - \sin x}{\cos x + \sin x}\,dx = \log|\sin x + \cos x| + C$

21. $\int \sin^{-1}(\cos x)\, dx$

Since $\cos x = \sin(\frac{\pi}{2} - x)$, we have $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$.

Sol. $= \int(\frac{\pi}{2} - x)\, dx = \frac{\pi x}{2} - \frac{x^2}{2} + C$

22. $\int \dfrac{1}{\cos(x-a)\cos(x-b)}\, dx$

Write $1 = \frac{\sin(a-b)}{\sin(a-b)}$ and $\sin[(x-b)-(x-a)] = \sin(a-b)$ to split into tangents.

Sol. $= \dfrac{1}{\sin(a-b)} \int[\tan(x-b) - \tan(x-a)]\, dx$

$= \dfrac{1}{\sin(a-b)} \log \left| \dfrac{\cos(x-a)}{\cos(x-b)} \right| + C$

23. $\int \dfrac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}\, dx$ **equals**

Split the fraction: $\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \sec^2 x - \operatorname{cosec}^2 x$.

Sol. $= \tan x + \cot x + C$

$\therefore$ Option (A)

24. $\int \dfrac{e^x(1+x)}{\cos^2(e^x x)}\, dx$ **equals**

Put $t = x e^x$, so $dt = e^x(1+x)\, dx$.

Sol. $= \int \sec^2 t\, dt = \tan(x e^x) + C$

$\therefore$ Option (B)