KUMAR CLASSES

NCERT SOLUTIONS · CLASS 12 MATHEMATICS

Exercise 7.2

Chapter 7 — Integrals · Integration by Substitution

Q1–37: integrate the given functions (mostly by substitution). Q38–39: choose the correct option.

1. $\int \frac{2x}{1+x^2}\,dx$

Put $t = 1 + x^2$, so $dt = 2x\,dx$; the numerator is exactly dt.

Sol. $= \int \frac{dt}{t} = \log(1 + x^2) + C$

2. $\int \frac{(\log x)^2}{x}\,dx$

Put $t = \log x$, so $dt = dx/x$.

Sol. $= \int t^2\,dt = \frac{(\log x)^3}{3} + C$

3. $\int \frac{1}{x + x\log x}\,dx$

Factor x: $x + x\log x = x(1 + \log x)$. Put $t = 1 + \log x$, $dt = dx/x$.

Sol. $= \int \frac{dt}{t} = \log|1 + \log x| + C$

4. $\int \sin x \sin(\cos x)\,dx$

Put $t = \cos x$, so $dt = -\sin x\,dx$.

Sol. $= -\int \sin t\,dt = \cos(\cos x) + C$

5. $\int \sin(ax+b)\cos(ax+b)\,dx$

Put $t = \sin(ax+b)$, so $dt = a\cos(ax+b)\,dx$.

Sol. $= \frac{1}{a}\int t\,dt = \frac{\sin^2(ax+b)}{2a} + C$

6. $\int \sqrt{ax+b}\,dx$

Standard power form after substitution $t = ax+b$.

Sol. $= \frac{2}{3a}(ax+b)^{3/2} + C$

7. $\int x\sqrt{x+2}\,dx$

Put $t = x+2$, so $x = t-2$ and $dx = dt$.

Sol. $= \int (t-2)\sqrt{t}\,dt = \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$

8. $\int x\sqrt{1+2x^2}\,dx$

Put $t = 1 + 2x^2$, so $dt = 4x\,dx$.

Sol. $= \frac{1}{4}\int \sqrt{t}\,dt = \frac{1}{6}(1+2x^2)^{3/2} + C$

9. $\int (4x+2)\sqrt{x^2+x+1}\,dx$

Note $4x+2 = 2(2x+1) = 2\cdot\frac{d}{dx}(x^2+x+1)$. Put $t = x^2+x+1$.

Sol. $= 2\int \sqrt{t}\,dt = \frac{4}{3}(x^2+x+1)^{3/2} + C$

10. $\int \frac{1}{x - \sqrt{x}}\,dx$

Write $x - \sqrt{x} = \sqrt{x}(\sqrt{x}-1)$ and put $u = \sqrt{x}$, $dx = 2u\,du$.

Sol. $= \int \frac{2}{u-1}\,du = 2\log|\sqrt{x} - 1| + C$

11. $\int \frac{x}{\sqrt{x+4}}\,dx$

Put $t = x+4$, so $x = t-4$.

Sol. $= \int (t^{1/2} - 4t^{-1/2})\,dt = \frac{2}{3}(x+4)^{3/2} - 8\sqrt{x+4} + C$

12. $\int x^5(x^3-1)^{1/3}\,dx$

Put $t = x^3-1$, so $x^3 = t+1$ and $x^2 dx = dt/3$; $x^5 dx = x^3\cdot x^2 dx$.

Sol. $= \frac{1}{3}\int (t+1)t^{1/3}\,dt = \frac{1}{7}(x^3-1)^{7/3} + \frac{1}{4}(x^3-1)^{4/3} + C$

13. $\int \frac{x^2}{(2+3x^3)^3}\,dx$

Put $t = 2 + 3x^3$, so $dt = 9x^2 dx$.

Sol. $= \frac{1}{9}\int t^{-3}\,dt = -\frac{1}{18(2+3x^3)^2} + C$

14. $\int \frac{1}{x(\log x)^m}\,dx$

Put $t = \log x$, $dt = dx/x$ ($m \neq 1$).

Sol. $= \int t^{-m}\,dt = \frac{(\log x)^{1-m}}{1-m} + C$

15. $\int \frac{x}{9-4x^2}\,dx$

Put $t = 9 - 4x^2$, so $dt = -8x\,dx$.

Sol. $= -\frac{1}{8}\int \frac{dt}{t} = -\frac{1}{8}\log|9-4x^2| + C$

16. $\int e^{2x+3}\,dx$

Linear exponent: divide by the coefficient of x.

Sol. $= \frac{1}{2}e^{2x+3} + C$

17. $\int \frac{x}{e^{x^2}}\,dx$

Rewrite as $x\,e^{-x^2}$; put $t = -x^2$, $dt = -2x\,dx$.

Sol. $= -\frac{1}{2}\int e^t\,dt = -\frac{1}{2}e^{-x^2} + C$

18. $\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$

Put $t = \tan^{-1}x$, so $dt = dx/(1+x^2)$.

Sol. $= \int e^t\,dt = e^{\tan^{-1}x} + C$

19. $\int \frac{e^{2x}-1}{e^{2x}+1}\,dx$

Divide num & den by e^x to get $\frac{e^x-e^{-x}}{e^x+e^{-x}}$; put $t = e^x+e^{-x}$.

Sol. $= \int \frac{dt}{t} = \log(e^x+e^{-x}) + C$

20. $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\,dx$

Put $t = e^{2x}+e^{-2x}$; then $dt = 2(e^{2x}-e^{-2x})dx$.

Sol. $= \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\log(e^{2x}+e^{-2x}) + C$

21. $\int \tan^2(2x-3)\,dx$

Use $\tan^2\theta = \sec^2\theta - 1$.

Sol. $= \int(\sec^2(2x-3)-1)\,dx = \frac{1}{2}\tan(2x-3) - x + C$

22. $\int \sec^2(7-4x)\,dx$

Standard integral with linear argument (coefficient −4).

Sol. $= -\frac{1}{4}\tan(7-4x) + C$

23. $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

Put $t = \sin^{-1}x$, so $dt = dx/\sqrt{1-x^2}$.

Sol. $= \int t\,dt = \frac{(\sin^{-1}x)^2}{2} + C$

24. $\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}\,dx$

The numerator is $\frac{1}{2}$ of $\frac{d}{dx}(6\cos x + 4\sin x) = 4\cos x - 6\sin x$.

Sol. $= \frac{1}{2}\log|6\cos x + 4\sin x| + C$

25. $\int \frac{1}{\cos^2x(1-\tan x)^2}\,dx$

Write as $\frac{\sec^2x}{(1-\tan x)^2}$; put $t = 1 - \tan x$, $dt = -\sec^2x\,dx$.

Sol. $= -\int t^{-2}\,dt = \frac{1}{1-\tan x} + C$

26. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

Put $t = \sqrt{x}$, so $dt = dx/(2\sqrt{x})$.

Sol. $= 2\int \cos t\,dt = 2\sin\sqrt{x} + C$

27. $\int \sqrt{\sin 2x}\,\cos 2x\,dx$

Put $t = \sin 2x$, so $dt = 2\cos 2x\,dx$.

Sol. $= \frac{1}{2}\int\sqrt{t}\,dt = \frac{1}{3}(\sin 2x)^{3/2} + C$

28. $\int \frac{\cos x}{\sqrt{1+\sin x}}\,dx$

Put $t = 1 + \sin x$, so $dt = \cos x\,dx$.

Sol. $= \int t^{-1/2}\,dt = 2\sqrt{1+\sin x} + C$

29. $\int \cot x \log(\sin x)\,dx$

Put $t = \log(\sin x)$, so $dt = \cot x\,dx$.

Sol. $= \int t\,dt = \frac{1}{2}(\log \sin x)^2 + C$

30. $\int \frac{\sin x}{1+\cos x}\,dx$

Put $t = 1 + \cos x$, so $dt = -\sin x\,dx$.

Sol. $= -\int \frac{dt}{t} = -\log|1+\cos x| + C$

31. $\int \frac{\sin x}{(1+\cos x)^2}\,dx$

Put $t = 1 + \cos x$, so $dt = -\sin x\,dx$.

Sol. $= -\int t^{-2}\,dt = \frac{1}{1+\cos x} + C$

32. $\int \frac{1}{1+\cot x}\,dx = \int \frac{\sin x}{\sin x + \cos x}\,dx$

With $J = \int \frac{\cos x}{\sin x+\cos x}\,dx$: $I+J = x$ and $J-I = \log|\sin x+\cos x|$.

Sol. $I = \frac{x}{2} - \frac{1}{2}\log|\sin x + \cos x| + C$

33. $\int \frac{1}{1-\tan x}\,dx = \int \frac{\cos x}{\cos x - \sin x}\,dx$

With $J = \int \frac{\sin x}{\cos x-\sin x}\,dx$: $I-J = x$ and $I+J = -\log|\cos x-\sin x|$.

Sol. $I = \frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + C$

34. $\int \frac{\sqrt{\tan x}}{\sin x \cos x}\,dx$

Since $\frac{1}{\sin x \cos x} = \frac{\sec^2 x}{\tan x}$, the integrand $= \frac{\sec^2 x}{\sqrt{\tan x}}$; put $t = \tan x$.

Sol. $= \int t^{-1/2}\,dt = 2\sqrt{\tan x} + C$

35. $\int \dfrac{(1+\log x)^2}{x}\, dx$

Put $t = 1 + \log x$, so $dt = dx/x$.

Sol. $= \int t^2\, dt = \dfrac{(1+\log x)^3}{3} + C$

---

36. $\int \dfrac{(x+1)(x+\log x)^2}{x}\, dx$

Since $\dfrac{x+1}{x} = 1 + \dfrac{1}{x} = \dfrac{d}{dx}(x+\log x)$, put $t = x + \log x$.

Sol. $= \int t^2\, dt = \dfrac{(x+\log x)^3}{3} + C$

---

37. $\int \dfrac{x^3 \sin(\tan^{-1}x^4)}{1+x^8}\, dx$

Put $t = \tan^{-1}(x^4)$, so $dt = \dfrac{4x^3}{1+x^8}\, dx$.

Sol. $= \dfrac{1}{4}\int \sin t\, dt = -\dfrac{1}{4} \cos(\tan^{-1}x^4) + C$

---

38. $\int \dfrac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x}\, dx$ equals

The numerator is exactly $\dfrac{d}{dx}(x^{10} + 10^x)$, so the integral is log of the denominator.

Sol. $= \log(10^x + x^{10}) + C$

$\therefore$ Option (D)

---

39. $\int \dfrac{dx}{\sin^2 x \cos^2 x}$ equals

Write $1 = \sin^2 x + \cos^2 x$ in the numerator and split.

Sol. $= \int(\sec^2 x + \mathrm{cosec}^2 x)\, dx = \tan x - \cot x + C$

$\therefore$ Option (B)