KUMAR CLASSES

NCERT SOLUTIONS · CLASS 12 MATHEMATICS

Exercise 7.1

Chapter 7 — Integrals

Q1–5: find an anti-derivative by inspection. Q6–20: evaluate the integrals. Q21–22: choose the correct option.

1. $\int \sin 2x \, dx$

Since $\frac{d}{dx}(\cos 2x) = -2 \sin 2x$, divide by -2.

Sol. $= -\frac{1}{2}\cos 2x + C$

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2. $\int \cos 3x \, dx$

As $\frac{d}{dx}(\sin 3x) = 3 \cos 3x$, divide by 3.

Sol. $= \frac{1}{3}\sin 3x + C$

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3. $\int e^{2x} \, dx$

Since $\frac{d}{dx}(e^{2x}) = 2e^{2x}$, divide by 2.

Sol. $= \frac{1}{2} e^{2x} + C$

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4. $\int (ax + b)^2 \, dx$

Substitute $ax + b = t$, so $a\,dx = dt$, then use the power rule.

Sol. $= \frac{1}{a}\int t^2 \, dt = \frac{t^3}{3a} = \frac{(ax+b)^3}{3a} + C$

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5. $\int (\sin 2x - 4e^{3x}) \, dx$

Integrate each term separately.

Sol. $= -\frac{1}{2}\cos 2x - \frac{4}{3} e^{3x} + C$

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6. $\int (4e^{3x} + 1)\,dx$

Integrate term by term; $\int e^{3x}dx = e^{3x}/3$.

Sol. $= \dfrac{4}{3} e^{3x} + x + C$

7. $\int x^2\left(1 - \dfrac{1}{x^2}\right) dx$

First simplify the integrand: $x^2(1 - 1/x^2) = x^2 - 1$.

Sol. $= \int (x^2 - 1)\,dx = \dfrac{x^3}{3} - x + C$

8. $\int (ax^2 + bx + c)\,dx$

Integrate each power of x using $\int x^n dx = x^{n+1}/(n+1)$.

Sol. $= \dfrac{ax^3}{3} + \dfrac{bx^2}{2} + cx + C$

9. $\int (2x^2 + e^x)\,dx$

Integrate term by term; $\int e^x dx = e^x$.

Sol. $= \dfrac{2}{3} x^3 + e^x + C$

10. $\int \left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right)^2 dx$

Expand the square: $(\sqrt{x} - 1/\sqrt{x})^2 = x - 2 + 1/x$, then integrate.

Sol. $= \int \left(x - 2 + \dfrac{1}{x}\right) dx = \dfrac{x^2}{2} - 2x + \log|x| + C$

11. $\int \dfrac{x^3 + 5x^2 - 4}{x^2}\,dx$

Divide each term of the numerator by x^2.

Sol. $= \int (x + 5 - 4x^{-2})\,dx = \dfrac{x^2}{2} + 5x + \dfrac{4}{x} + C$

12. $\int \dfrac{x^3 + 3x + 4}{\sqrt{x}}\,dx$

Divide each term by $\sqrt{x} = x^{1/2}$ to get powers of x, then apply the power rule.

Sol. $= \int (x^{5/2} + 3x^{1/2} + 4x^{-1/2})\,dx$

$= \dfrac{2}{7} x^{7/2} + 2x^{3/2} + 8\sqrt{x} + C$

13. $\int \dfrac{x^3 - x^2 + x - 1}{x - 1}\, dx$

Factorise the numerator and cancel $(x - 1)$: $x^3-x^2+x-1 = (x-1)(x^2+1)$.

Sol. $= \int (x^2 + 1)\, dx = \dfrac{x^3}{3} + x + C$

14. $\int (1 - x)\sqrt{x}\, dx$

Multiply out: $(1 - x)\sqrt{x} = x^{1/2} - x^{3/2}$.

Sol. $= \int (x^{1/2} - x^{3/2})\, dx = \dfrac{2}{3} x^{3/2} - \dfrac{2}{5} x^{5/2} + C$

15. $\int \sqrt{x}\,(3x^2 + 2x + 3)\, dx$

Multiply $\sqrt{x}$ through the bracket to get powers of x.

Sol. $= \int (3x^{5/2} + 2x^{3/2} + 3x^{1/2})\, dx$

$= \dfrac{6}{7} x^{7/2} + \dfrac{4}{5} x^{5/2} + 2x^{3/2} + C$

16. $\int (2x - 3\cos x + e^x)\, dx$

Integrate each term; $\int \cos x\, dx = \sin x$, $\int e^x dx = e^x$.

Sol. $= x^2 - 3\sin x + e^x + C$

17. $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

Integrate term by term; $\int \sin x\, dx = -\cos x$, so $-3\int \sin x = +3\cos x$.

Sol. $= \dfrac{2}{3} x^3 + 3\cos x + \dfrac{10}{3} x^{3/2} + C$

18. $\int \sec x\,(\sec x + \tan x)\, dx$

Expand first: $\sec x(\sec x + \tan x) = \sec^2 x + \sec x \tan x$ (standard integrals).

Sol. $= \int (\sec^2 x + \sec x \tan x)\, dx = \tan x + \sec x + C$

19. $\int \dfrac{\sec^2 x}{\operatorname{cosec}^2 x}\, dx$

Write $\dfrac{\sec^2 x}{\operatorname{cosec}^2 x} = \dfrac{\sin^2 x}{\cos^2 x} = \tan^2 x = \sec^2 x - 1$.

Sol. $= \int (\sec^2 x - 1)\, dx = \tan x - x + C$

20. $\int \frac{2 - 3\sin x}{\cos^2 x}\, dx$

Split the fraction: $\frac{2}{\cos^2 x} = 2\sec^2 x$ and $\frac{\sin x}{\cos^2 x} = \sec x \tan x$.

Sol. $= \int (2\sec^2 x - 3\sec x \tan x)\, dx = 2\tan x - 3\sec x + C$

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21. **The anti-derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ equals**

Write as $x^{1/2} + x^{-1/2}$ and use the power rule on each term.

Sol. $\int (x^{1/2} + x^{-1/2})\, dx = \frac{2}{3} x^{3/2} + 2x^{1/2} + C$

$\therefore$ Option (C)

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22. **If $\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$ with $f(2) = 0$, then $f(x)$ is**

Integrate to get $f(x)$ with an arbitrary constant C, then use $f(2) = 0$ to find C.

Sol. $f(x) = \int (4x^3 - 3x^{-4})\, dx = x^4 + \frac{1}{x^3} + C$

$f(2) = 16 + \frac{1}{8} + C = 0 \Rightarrow C = -\frac{129}{8}$

$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$

$\therefore$ Option (A)