KUMAR CLASSES

Excellence in Mathematics

CLASS 10 MATHEMATICS

Chapter 1 : REAL NUMBERS – NCERT SOLUTIONS

Complete Concept Notes

Full Concepts • Solved Examples • Board Questions • Concept Check

Strong Concepts + Regular Practice = High Score

Exercise 1.1

Q1. Express each number as a product of its prime factors: (i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429.

Solution :

- *(i)* $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7.$
- *(ii)* $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13.$
- *(iii)* $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17.$
- *(iv)* $5005 = 5 \times 7 \times 11 \times 13.$
- *(v)* $7429 = 17 \times 19 \times 23.$

Q2. Find the LCM and HCF of the following pairs of integers and verify that $\mathrm{LCM} \times \mathrm{HCF} =$ product of the two numbers: (i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54.

Solution :

- *(i)* $26 = 2 \times 13,\ 91 = 7 \times 13$. *HCF* $= 13$, *LCM* $= 2 \times 7 \times 13 = 182$.
- **Verify:** $13 \times 182 = 2366 = 26 \times 91$.
- *(ii)* $510 = 2 \times 3 \times 5 \times 17,\ 92 = 2^2 \times 23$. *HCF* $= 2$, *LCM* $= 2^2 \times 3 \times 5 \times 17 \times 23 = 23460$.
- **Verify:** $2 \times 23460 = 46920 = 510 \times 92$.
- *(iii)* $336 = 2^4 \times 3 \times 7,\ 54 = 2 \times 3^3$. *HCF* $= 2 \times 3 = 6$, *LCM* $= 2^4 \times 3^3 \times 7 = 3024$.
- **Verify:** $6 \times 3024 = 18144 = 336 \times 54$.

Q3. Find the LCM and HCF of the following integers by applying the prime factorisation method: (i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25.

Solution :

- *(i)* $12 = 2^2 \times 3,\ 15 = 3 \times 5,\ 21 = 3 \times 7$. *HCF* $= 3$; *LCM* $= 2^2 \times 3 \times 5 \times 7 = 420$.
- *(ii)* $17, 23, 29$ *are all prime*. *HCF* $= 1$; *LCM* $= 17 \times 23 \times 29 = 11339$.
- *(iii)* $8 = 2^3,\ 9 = 3^2,\ 25 = 5^2$. *HCF* $= 1$; *LCM* $= 2^3 \times 3^2 \times 5^2 = 1800$.

Q4. Given that HCF$(306, 657) = 9$, find LCM$(306, 657)$.

Solution :

- $\text{LCM} = \dfrac{306 \times 657}{\text{HCF}} = \dfrac{306 \times 657}{9} = \dfrac{201042}{9} = 22338.$

Answer : LCM$(306, 657) = 22338$

Q5. Check whether 6^n can end with the digit 0 for any natural number n.

Solution :

- *A number ends in 0 only if it is divisible by $10 = 2 \times 5$, i.e. it must have both 2 and 5 as prime factors.*
- $6^n = (2 \times 3)^n = 2^n \times 3^n$, *whose only prime factors are 2 and 3.*
- *Since 5 is not a factor of 6^n, it can never end with the digit 0.*

Answer : 6^n can never end in 0

Q6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Solution :

- $7 \times 11 \times 13 + 13 = 13\,(7 \times 11 + 1) = 13 \times 78$. *It has a factor* 13 *(other than* 1 *and itself)* $\Rightarrow$ *composite.*
- $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5\,(7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) = 5 \times 1009$. *It has a factor* $5 \Rightarrow$ *composite.*

Q7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round, while Ravi takes 12 minutes. If they both start at the same point and time and go in the same direction, after how many minutes will they meet again at the starting point?

Solution :

- *They meet again at the starting point after a time equal to LCM*$(18, 12)$ *minutes.*
- $18 = 2 \times 3^2,\ 12 = 2^2 \times 3 \Rightarrow \text{LCM} = 2^2 \times 3^2 = 36.$
- $\therefore$ *They will meet again after* 36 *minutes.*

Answer : After 36 minutes

Exercise 1.2

Q1. Prove that $\sqrt{5}$ is irrational.

Solution :

- **Proof (by contradiction).** *Suppose* $\sqrt{5}$ *is rational.*
- *Then* $\sqrt{5} = \dfrac{a}{b}$ *for some coprime integers* a, b $(b \neq 0)$.
- *Squaring:* $5 = \dfrac{a^2}{b^2} \Rightarrow a^2 = 5b^2$ *...(1). So* $5 \mid a^2$, *hence* $5 \mid a$. *Let* $a = 5c$.
- *Substituting in (1):* $25c^2 = 5b^2 \Rightarrow b^2 = 5c^2$. *So* $5 \mid b^2$, *hence* $5 \mid b$.
- *Then* 5 *divides both* a *and* b, *contradicting that they are coprime.*
- *Hence our assumption is wrong, and* $\sqrt{5}$ *is irrational.* **(Proved)**

Q2. Prove that $3 + 2\sqrt{5}$ is irrational.

Solution :

- **Proof.** *Suppose* $3 + 2\sqrt{5}$ *is rational, say* $3 + 2\sqrt{5} = r$ *where* r *is rational.*
- *Then* $2\sqrt{5} = r - 3 \Rightarrow \sqrt{5} = \dfrac{r-3}{2}$.
- *Since* r *is rational,* $\dfrac{r-3}{2}$ *is rational, so* $\sqrt{5}$ *would be rational.*
- *But* $\sqrt{5}$ *is irrational (proved above) — a contradiction.*
- *Hence* $3 + 2\sqrt{5}$ *is irrational.* **(Proved)**

Q3. Prove that the following are irrational: (i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6+\sqrt{2}$.

Solution :

- **(i)** *Suppose* $\frac{1}{\sqrt{2}}$ *is rational. Then* $\sqrt{2} = \frac{1}{(1/\sqrt{2})}$ *would be rational — but* $\sqrt{2}$ *is irrational. Contradiction, so* $\frac{1}{\sqrt{2}}$ *is irrational.*
- **(ii)** *Suppose* $7\sqrt{5} = r$ *(rational). Then* $\sqrt{5} = \frac{r}{7}$ *is rational — contradiction. So* $7\sqrt{5}$ *is irrational.*
- **(iii)** *Suppose* $6+\sqrt{2} = r$ *(rational). Then* $\sqrt{2} = r-6$ *is rational — contradiction. So* $6+\sqrt{2}$ *is irrational.* **(Proved)**

Concept Check — Test Your Understanding

Try each one in your own words first, then check the answer below.

Q1. What does the Fundamental Theorem of Arithmetic guarantee about a composite number?

Answer: *Every composite number can be written as a product of primes, and this prime factorisation is* ***unique****, apart from the order of the factors.*

Q2. Can two numbers have HCF $=15$ and LCM $=175$? Justify.

Answer: *No. The HCF always divides the LCM, but 15 does not divide 175. So such a pair cannot exist.*

Q3. If $HCF(a,b)=1$, what is $LCM(a,b)$?

Answer: *The numbers are co-prime, so $LCM(a,b)=a\times b$ (since HCF $\times$ LCM $=a\times b$).*

Q4. Why can 6^n never end with the digit 0?

Answer: *A number ends in 0 only if $10=2\times5$ divides it. But $6^n=2^n\times3^n$ has no factor 5, so it can never end in 0.*

Q5. Which method is used to prove that $\sqrt{p}$ (p prime) is irrational?

Answer: *Proof by contradiction: assume $\sqrt{p}=a/b$ with a,b coprime; it forces p to divide both a and b, contradicting coprimeness.*

Q6. Is the product of two irrational numbers always irrational?

Answer: *No. For example $\sqrt{2}\times\sqrt{2}=2$, which is rational.*

Q7. In the prime factorisation of a perfect square, what is special about the powers of the primes?

Answer: *Every prime appears to an* ***even*** *power (e.g. $36=2^2\times3^2$).*

Q8. Does HCF $\times$ LCM $=$ product hold for three numbers as well?

Answer: *No. It holds only for two numbers. For three numbers HCF$\times$LCM is generally not equal to their product.*