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CLASS 10 MATHEMATICS

Chapter 3 : PAIR OF LINEAR EQUATIONS IN TWO VARIABLES – NCERT SOLUTIONS

Complete Concept Notes

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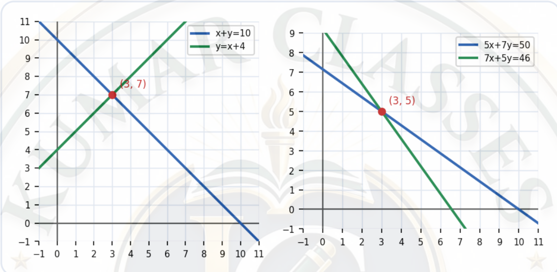
Exercise 3.1

Q1. Form the pair of linear equations in the following problems, and find their solutions graphically. (i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls. (ii) 5 pencils and 7 pens together cost ₹50, whereas 7 pencils and 5 pens together cost ₹46. Find the cost of one pencil and one pen.



Solution :

- **(i)** *Let boys* $= x$, *girls* $= y$. *Then* $x + y = 10$ *and* $y = x + 4$.
- *The lines meet at* $(3, 7)$, *so* $x = 3,\ y = 7$: $\mathbf{Boys = 3,\ Girls = 7}$.
- **(ii)** *Let pencil* $=$ ₹x, *pen* $=$ ₹y. *Then* $5x + 7y = 50$ *and* $7x + 5y = 46$.
- *The lines meet at* $(3, 5)$, *so* **Pencil = ₹3, Pen = ₹5**.

Answer : Boys 3, Girls 7; Pencil ₹3, Pen ₹5

Q2. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$, $\frac{c_1}{c_2}$, find out whether the lines are intersecting, parallel or coincident: (i) $5x - 4y + 8 = 0,\ 7x + 6y - 9 = 0$ (ii) $9x + 3y + 12 = 0,\ 18x + 6y + 24 = 0$ (iii) $6x - 3y + 10 = 0,\ 2x - y + 9 = 0$.

Solution :

TIP $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ intersecting (one solution); $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ parallel (no solution); $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow$ coincident (infinite solutions).

- **(i)** $\frac{a_1}{a_2} = \frac{5}{7}$, $\frac{b_1}{b_2} = \frac{-4}{6} = -\frac{2}{3}$. Since $\frac{5}{7} \neq -\frac{2}{3}$, the lines are **intersecting**.
- **(ii)** $\frac{9}{18} = \frac{3}{6} = \frac{12}{24} = \frac{1}{2}$. All equal $\Rightarrow$ the lines are **coincident**.
- **(iii)** $\frac{6}{2} = \frac{-3}{-1} = 3$ but $\frac{10}{9} \neq 3$. So $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ the lines are **parallel**.

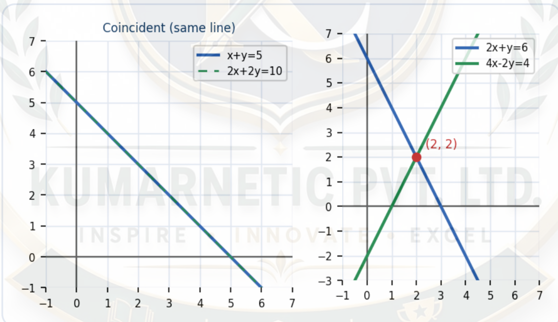
Q3. On comparing the ratios, find out whether the following pairs of linear equations are consistent or inconsistent: (i) $3x + 2y = 5,\ 2x - 3y = 7$ (ii) $2x - 3y = 8,\ 4x - 6y = 9$ (iii) $\frac{3}{2}x + \frac{5}{3}y = 7,\ 9x - 10y = 14$ (iv) $5x - 3y = 11,\ -10x + 6y = -22$ (v) $\frac{4}{3}x + 2y = 8,\ 2x + 3y = 12$.

Solution :

- **(i)** $\frac{3}{2} \neq \frac{2}{-3} \Rightarrow$ intersecting $\Rightarrow$ **consistent** (unique solution).

- **(ii)** $\frac{2}{4} = \frac{-3}{-6} = \frac{1}{2} \neq \frac{8}{9} \Rightarrow$ *parallel* $\Rightarrow$ **inconsistent**.
- **(iii)** $\frac{3/2}{9} = \frac{1}{6}, \frac{5/3}{-10} = -\frac{1}{6}$; *unequal* $\Rightarrow$ *intersecting* $\Rightarrow$ **consistent**.
- **(iv)** $\frac{5}{-10} = \frac{-3}{6} = \frac{11}{-22} = -\frac{1}{2} \Rightarrow$ *coincident* $\Rightarrow$ **consistent** *(infinite solutions)*.
- **(v)** $\frac{4/3}{2} = \frac{2}{3}, \frac{2}{3}, \frac{8}{12} = \frac{2}{3}$; *all equal* $\Rightarrow$ *coincident* $\Rightarrow$ **consistent**.

Q4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically: (i) $x + y = 5,\ 2x + 2y = 10$ **(ii)** $x - y = 8,\ 3x - 3y = 16$ **(iii)** $2x + y - 6 = 0,\ 4x - 2y - 4 = 0$ **(iv)** $2x - 2y - 2 = 0,\ 4x - 4y - 5 = 0.$



Solution :

- **(i)** $\frac{1}{2} = \frac{1}{2} = \frac{5}{10} \Rightarrow$ *coincident* $\Rightarrow$ **consistent**, *infinitely many solutions (same line)*.
- **(ii)** $\frac{1}{3} = \frac{-1}{-3} = \frac{1}{3} \neq \frac{8}{16} \Rightarrow$ *parallel* $\Rightarrow$ **inconsistent** *(no solution)*.
- **(iii)** $\frac{2}{4} \neq \frac{1}{-2} \Rightarrow$ *intersecting* $\Rightarrow$ **consistent**. *Solving:* $x = 2,\ y = 2$.
- **(iv)** $\frac{2}{4} = \frac{-2}{-4} = \frac{1}{2} \neq \frac{-2}{-5} \Rightarrow$ *parallel* $\Rightarrow$ **inconsistent**.

Answer : (i) consistent (infinite) (ii) inconsistent (iii) consistent, $(2, 2)$ (iv) inconsistent

Q5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Solution :

- *Let length* $= x$, *width* $= y$. *Then* $x = y + 4$ *and* $x + y = 36$ *(half the perimeter).*
- *Substituting:* $(y + 4) + y = 36 \Rightarrow 2y = 32 \Rightarrow y = 16$, *and* $x = 20$.

Answer : Length $= 20$ m, Width $= 16$ m

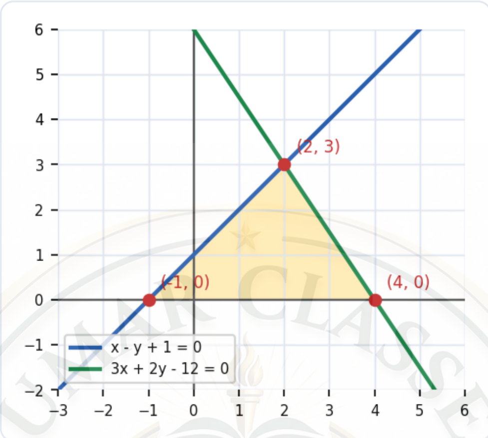
Q6. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair is (i) intersecting lines (ii) parallel lines (iii) coincident lines.

Solution :

TIP *Compare with* $2x + 3y - 8 = 0$: *intersecting needs* $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$; *parallel needs* $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$; *coincident needs all three equal.*

- **(i) Intersecting:** *e.g.* $3x - 2y - 4 = 0$ *(since* $\frac{2}{3} \neq \frac{3}{-2}$*).*
- **(ii) Parallel:** *e.g.* $2x + 3y - 12 = 0$ *(same* a, b, *different* c*).*
- **(iii) Coincident:** *e.g.* $4x + 6y - 16 = 0$ *(exactly* $2\times$ *the given equation).*

Q7. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.



Solution :

- *The two lines meet where* $x - y + 1 = 0$ *and* $3x + 2y - 12 = 0$*: solving gives* $(2, 3)$.
- $x - y + 1 = 0$ *meets the* x*-axis* $(y = 0)$ *at* $(-1, 0)$*;* $3x + 2y - 12 = 0$ *meets it at* $(4, 0)$.
- *So the triangle has vertices* $(-1, 0)$, $(4, 0)$ *and* $(2, 3)$ *(shaded region).*

Answer : Vertices: $(-1, 0)$, $(4, 0)$, $(2, 3)$

Exercise 3.2

Q1. Solve the following pairs of linear equations by the substitution method.

Solution :

- **(i)** $x + y = 14,\ x - y = 4$: *from the 2nd,* $x = y + 4$. *Then* $(y + 4) + y = 14 \Rightarrow y = 5,\ x = 9$.
- **(ii)** $s - t = 3,\ \frac{s}{3} + \frac{t}{2} = 6$: $s = t + 3 \Rightarrow \frac{t+3}{3} + \frac{t}{2} = 6 \Rightarrow 2(t + 3) + 3t = 36 \Rightarrow t = 6,\ s = 9$.
- **(iii)** $3x - y = 3,\ 9x - 3y = 9$: *the 2nd is* $3\times$ *the Ist, so they are the same line* $\Rightarrow$ **infinitely many solutions**.
- **(iv)** $0.2x + 0.3y = 1.3,\ 0.4x + 0.5y = 2.3$ *i.e.* $2x + 3y = 13,\ 4x + 5y = 23$. *Substituting gives* $x = 2,\ y = 3$.
- **(v)** $\sqrt{2}\,x + \sqrt{3}\,y = 0,\ \sqrt{3}\,x - \sqrt{8}\,y = 0$: *from the Ist* $x = -\frac{\sqrt{3}}{\sqrt{2}}y$; *substituting gives* $y = 0$, *hence* $x = 0$.
- **(vi)** $\frac{3x}{2} - \frac{5y}{3} = -2,\ \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$ *i.e.* $9x - 10y = -12,\ 2x + 3y = 13$. *Solving gives* $x = 2,\ y = 3$.

Answer : (i) $9, 5$ (ii) $s = 9, t = 6$ (iii) infinite (iv) $2, 3$ (v) $0, 0$ (vi) $2, 3$

Q2. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of m for which $y = mx + 3$.

Solution :

- *Subtracting:* $(2x+3y)-(2x-4y)=11-(-24)\Rightarrow 7y=35\Rightarrow y=5$.
- *Then* $2x+15=11\Rightarrow x=-2$.
- *Put in* $y=mx+3$: $5=-2m+3\Rightarrow m=-1$.

Answer : $x=-2,\ y=5,\ m=-1$

Q3. Form the pair of linear equations for the following and solve by substitution. (i) The difference between two numbers is 26 and one is three times the other. (ii) The larger of two supplementary angles exceeds the smaller by 18°. (iii) 7 bats and 6 balls cost ₹3800; 3 bats and 5 balls cost ₹1750. Find the cost of each. (iv) Taxi charge for 10 km is ₹105 and for 15 km is ₹155. Find the fixed charge and per-km charge, and the charge for 25 km. (v) A fraction becomes $\frac{9}{11}$ if 2 is added to both numerator and denominator, and $\frac{5}{6}$ if 3 is added to both. Find it.

Solution :

- **(i)** $x-y=26,\ x=3y\Rightarrow 3y-y=26\Rightarrow y=13,\ x=39$. *Numbers* $\mathbf{39,\ 13}$.
- **(ii)** $x+y=180,\ x-y=18\Rightarrow x=99,\ y=81$. *Angles* $\mathbf{99^\circ,\ 81^\circ}$.
- **(iii)** $7x+6y=3800,\ 3x+5y=1750\Rightarrow x=500,\ y=50$. *Bat* **₹500**, *Ball* **₹50**.
- **(iv)** *Fixed* f, *per-km* d: $f+10d=105,\ f+15d=155\Rightarrow d=10,\ f=5$. *For* 25 *km:* $5+25(10)=$ ₹255.

· **(v)**

$\frac{x+2}{y+2} = \frac{9}{11},\ \frac{x+3}{y+3} = \frac{5}{6} \Rightarrow 11x - 9y = -4,\ 6x - 5y = -3 \Rightarrow x = 7,\ y = 9$

. *Fraction* $\mathbf{\frac{7}{9}}$.

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Exercise 3.3

Q1. Solve the following pairs of linear equations by the elimination method and the substitution method: (i) $x + y = 5$ and $2x - 3y = 4$ (ii) $3x + 4y = 10$ and $2x - 2y = 2$ (iii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$ (iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$.

Solution :

- **(i)** *From* $x + y = 5,\ y = 5 - x$. *Then*

  $2x - 3(5 - x) = 4 \Rightarrow 5x = 19 \Rightarrow x = \frac{19}{5},\ y = \frac{6}{5}$.
- **(ii)** $2x - 2y = 2 \Rightarrow x - y = 1$. *Then*

  $3(y + 1) + 4y = 10 \Rightarrow 7y = 7 \Rightarrow y = 1,\ x = 2$.
- **(iii)** $3x - 5y = 4,\ 9x - 2y = 7$. *Multiply Ist by* 3: $9x - 15y = 12$; *subtract:*

  $-13y = 5 \Rightarrow y = -\frac{5}{13},\ x = \frac{9}{13}$.
- **(iv)** $3x + 4y = -6,\ 3x - y = 9$. *Subtract:* $5y = -15 \Rightarrow y = -3,\ x = 2$.

Answer : (i) $\frac{19}{5}, \frac{6}{5}$ (ii) $2, 1$ (iii) $\frac{9}{13}, -\frac{5}{13}$ (iv) $2, -3$

Q2. Form the pair of linear equations and find their solutions (if they exist) by the elimination method: (i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1; it becomes $\frac{1}{2}$ if we only add 1 to the denominator. Find the fraction. (ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. Find their ages. (iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the digits. Find the number.

Solution :

- **(i)** $\frac{x+1}{y-1} = 1 \Rightarrow x - y = -2;\ \frac{x}{y+1} = \frac{1}{2} \Rightarrow 2x - y = 1$. *Eliminating:* $x = 3,\ y = 5$. *Fraction* $\mathbf{\frac{3}{5}}$.
- **(ii)** $n - 5 = 3(s-5)$ *and* $n + 10 = 2(s+10) \Rightarrow n = 3s - 10,\ n = 2s + 10 \Rightarrow s = 20,\ n = 50$. *Nuri* $\mathbf{50}$, *Sonu* $\mathbf{20}$.
- **(iii)** $x + y = 9$ *and* $9(10x + y) = 2(10y + x) \Rightarrow 8x - y = 0$. *Solving:* $x = 1,\ y = 8$. *Number* $\mathbf{18}$.

Concept Check — Test Your Understanding

Try each one in your own words first, then check the answer below.

Q1. When is a pair of linear equations called consistent?

Answer: When it has at least one solution — i.e. the lines intersect at a point (unique solution) or are coincident (infinitely many solutions).

Q2. How do the ratios $(a_1)/(a_2),(b_1)/(b_2),(c_1)/(c_2)$ decide the type of lines?

Answer: $(a_1)/(a_2)\neq(b_1)/(b_2)$: intersecting (unique). $(a_1)/(a_2)=(b_1)/(b_2)\neq(c_1)/(c_2)$: parallel (no solution). $(a_1)/(a_2)=(b_1)/(b_2)=(c_1)/(c_2)$: coincident (infinite).

Q3. What does it mean graphically if a pair of equations is inconsistent?

Answer: The two lines are parallel and never meet, so there is no common solution.

Q4. If two lines are coincident, how many solutions does the pair have?

Answer: Infinitely many — every point on the line is a solution.

Q5. Which methods can be used to solve a pair of linear equations algebraically?

Answer: Substitution and elimination (and, graphically, by drawing the two lines).

Q6. For $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ to have a unique solution, what must hold?

Answer: $(a_1)/(a_2)\neq(b_1)/(b_2)$ (the lines are intersecting).

Q7. In a word problem, what is the first step in forming the equations?

Answer: Choose variables for the unknowns, then translate each given condition into a linear equation.

Q8. How do you check a solution of a pair of linear equations?

Answer: Substitute the values into both original equations; the solution is correct if both are satisfied.