



CLASS 10 MATHS

PART Ex 5.3 · Examples

ARITHMETIC PROGRESSIONS

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NCERT SOLVED EXAMPLES

E11 Find the sum of the first 22 terms of the AP : 8, 3, -2, ...

$$a = 8, d = 3 - 8 = -5, n = 22$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{22} = \frac{22}{2}[2(8) + 21(-5)]$$

$$S_{22} = 11(16 - 105) = 11(-89) = -979$$

E12 If the sum of the first 14 terms of an AP is 1050 and $a = 10$, find the 20th term.

$$1050 = \frac{14}{2}[2(10) + 13d]$$

$$1050 = 140 + 91d \Rightarrow 91d = 910 \Rightarrow d = 10$$

$$a_{20} = 10 + (20-1)(10) = 200$$

E13 How many terms of the AP : 24, 21, 18, ... give a sum of 78?

$$a = 24, d = -3, S_n = 78$$

$$78 = \frac{n}{2}[48 + (n-1)(-3)]$$

$$n^2 - 17n + 52 = 0$$

$$(n-4)(n-13) = 0 \quad n = 4 \text{ or } 13$$

Both are valid — terms 5 to 13 add up to zero.

EXAMPLES (Continued)

E14 Find the sum of (i) the first 1000 positive integers (ii) the first n positive integers.

$$(i) S = \frac{n}{2}(a + \ell) = \frac{1000}{2}(1 + 1000)$$

$$S = 500 \times 1001 = 500500$$

$$(ii) S_n = \frac{n}{2}(1 + n) = \frac{n(n+1)}{2}$$

E15 Find the sum of the first 24 terms whose n^{th} term is $a_n = 3 + 2n$.

$$a_1 = 3 + 2 = 5, a_2 = 7 \rightarrow d = 2$$

$$S_{24} = \frac{24}{2}[2(5) + 23(2)]$$

$$S_{24} = 12(10 + 46) = 12 \times 56 = 672$$

E16 A TV maker produced 600 sets in the 3rd year and 700 in the 7th year. Find (i) 1st-year, (ii) 10th-year, (iii) total in first 7 years.

$$a + 2d = 600, a + 6d = 700 \Rightarrow 4d = 100 \Rightarrow d = 25$$

$$a = 600 - 50 = 550 \quad (i) 550$$

$$a_{10} = 550 + 9(25) = 775 \quad (ii) 775$$

$$S_7 = \frac{7}{2}[1100 + 150] = 7 \times 625 = 4375 \quad (iii) 4375$$

★ **Remember :** Sum of first n terms : $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + \ell)$, where ℓ is the last term.

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PART 1 (Ex 5.3)

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EXERCISE 5.3 — SOLUTIONS

1 Find the sum of the following APs.

(i) 2, 7, 12, ... to 10 terms

$$a = 2, d = 5, n = 10$$

$$S_{10} = \frac{10}{2} [2(2) + 9(5)]$$

$$S_{10} = 5(4 + 45) = 5 \times 49 = 245$$

(ii) -37, -33, -29, ... to 12 terms

$$a = -37, d = 4, n = 12$$

$$S_{12} = \frac{12}{2} [2(-37) + 11(4)]$$

$$S_{12} = 6(-74 + 44) = 6(-30) = -180$$



EXERCISE 5.3 — SOLUTIONS (Continued)

1 (Continued)

(iii) 0.6, 1.7, 2.8, ... to 100 terms

$$a = 0.6, d = 1.1, n = 100$$

$$S_{100} = \frac{100}{2} [2(0.6) + 99(1.1)]$$

$$S_{100} = 50(1.2 + 108.9) = 50 \times 110.1 = 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$ to 11 terms

$$a = \frac{1}{15}, d = \frac{1}{60}, n = 11$$

$$S_{11} = \frac{11}{2} \left[-\frac{2}{15} + 10\left(\frac{1}{60}\right) \right]$$

$$S_{11} = \frac{11}{2} \times \frac{3}{10} = \frac{33}{20}$$

**Remember :** Identify a , d and n , then substitute in $S_n = \frac{n}{2} [2a + (n - 1)d]$.

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PART 2 (Ex 5.3)

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EXERCISE 5.3 — SOLUTIONS (Continued)

2 Find the sums given below.

(i) $7 + 10\frac{1}{2} + 14 + \dots + 84$

$a = 7, d = 3.5, \ell = 84$

$84 = 7 + (n - 1)(3.5) \Rightarrow n = 23$

$S = \frac{23}{2}(7 + 84) = \frac{23}{2}(91) = 1046\frac{1}{2}$

(ii) $34 + 32 + 30 + \dots + 10$

$a = 34, d = -2, \ell = 10 \Rightarrow n = 13$

$S = \frac{13}{2}(34 + 10) = 13 \times 22 = 286$

(iii) $-5 + (-8) + \dots + (-230)$

$a = -5, d = -3, \ell = -230 \Rightarrow n = 76$

$S = \frac{76}{2}(-5 - 230) = 38(-235) = -8930$

EXERCISE 5.3 — SOLUTIONS (Continued)

3 In an AP :

(i) $a = 5, d = 3, a_n = 50$

$50 = 5 + (n - 1)(3) \Rightarrow n = 16$

$S_n = \frac{16}{2}(5 + 50) = 8 \times 55 = 440$

(ii) $a = 7, a_{13} = 35$

$7 + 12d = 35 \Rightarrow d = \frac{7}{3}$

$S_{13} = \frac{13}{2}(7 + 35) = 13 \times 21 = 273$

(iii) $a_{12} = 37, d = 3$

$a = 37 - 33 = 4$

$S_{12} = \frac{12}{2}(4 + 37) = 6 \times 41 = 246$

(iv) $a_3 = 15, S_{10} = 125$

$a + 2d = 15; 2a + 9d = 25$

Solving : $d = -1, a = 17$

(v) $d = 5, S_9 = 75$

$2a + 40 = \frac{50}{3} \Rightarrow a = -\frac{35}{3}, a_9 = \frac{85}{3}$

**Remember :** When the last term ℓ is known, use $S_n = \frac{n}{2}(a + \ell)$.

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EXERCISE 5.3 — SOLUTIONS (Continued)

3 (Continued)

(vi) $a = 2, d = 8, S_n = 90$

$$90 = \frac{n}{2}[4 + 8(n - 1)] = n(4n - 2)$$

$$4n^2 - 2n - 90 = 0 \Rightarrow 2n^2 - n - 45 = 0$$

$$(2n + 9)(n - 5) = 0 \Rightarrow n = 5$$

$$a_n = 2 + 4(8) \quad a_n = 34$$

(vii) $a = 8, a_n = 62, S_n = 210$

$$210 = \frac{n}{2}(8 + 62) = 35n \Rightarrow n = 6$$

$$8 + 5d = 62 \quad d = \frac{54}{5}$$

(viii) $a_n = 4, d = 2, S_n = -14$

$$S_n = n(5 - n) = -14 \Rightarrow n^2 - 5n - 14 = 0$$

$$(n - 7)(n + 2) = 0 \Rightarrow n = 7, a = 6 - 14 \quad a = -8$$

EXERCISE 5.3 — SOLUTIONS (Continued)

3 (Continued)

(ix) $a = 3, n = 8, S = 192$

$$192 = \frac{8}{2}[2(3) + 7d] = 4(6 + 7d)$$

$$48 = 6 + 7d \Rightarrow 7d = 42 \quad d = 6$$

(x) $l = 28, S = 144, n = 9$

$$144 = \frac{9}{2}(a + 28)$$

$$a + 28 = 32 \quad a = 4$$

4 How many terms of the AP : 9, 17, 25, ... must be taken to give a sum of 636?

 $a = 9, d = 8, S_n = 636$

$$636 = \frac{n}{2}[18 + 8(n - 1)]$$

$$1272 = n(8n + 10) \Rightarrow 4n^2 + 5n - 636 = 0$$

$$(4n + 53)(n - 12) = 0 \quad n = 12$$

5 First term 5, last term 45, sum 400. Find n and d .

$$S = \frac{n}{2}(5 + 45) = 25n = 400 \Rightarrow n = 16$$

$$45 = 5 + 15d \Rightarrow 15d = 40 \quad d = \frac{8}{3}$$



Remember :

To find how many terms give a sum, form a quadratic in n and keep the positive whole-number root.

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PART 4 (Ex 5.3)

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EXERCISE 5.3 — SOLUTIONS (Continued)

- 6 First and last terms are 17 and 350, $d = 9$. Find n and the sum.

$$350 = 17 + (n - 1)(9)$$

$$333 = 9(n - 1) \Rightarrow n - 1 = 37 \quad \boxed{n = 38}$$

$$S = \frac{38}{2}(17 + 350) = 19 \times 367 \quad \boxed{S = 6973}$$

- 7 Find the sum of first 22 terms where $d = 7$ and the 22nd term is 149.

$$a_{22} = a + 21d = 149 \Rightarrow a + 147 = 149 \Rightarrow a = 2$$

$$S_{22} = \frac{22}{2}(2 + 149) = 11 \times 151 \quad \boxed{= 1661}$$

- 8 Find the sum of first 51 terms whose 2nd and 3rd terms are 14 and 18.

$$d = 18 - 14 = 4, \quad a = 14 - 4 = 10$$

$$S_{51} = \frac{51}{2}[2(10) + 50(4)]$$

$$S_{51} = \frac{51}{2}(220) = 51 \times 110 \quad \boxed{= 5610}$$

- 9 Sum of first 7 terms is 49 and of 17 terms is 289. Find the sum of first n terms.

$$a + 3d = 7 \quad \dots\dots(i)$$

$$a + 8d = 17 \quad \dots\dots(ii)$$

$$\text{Subtracting : } 5d = 10 \Rightarrow d = 2, \quad a = 1$$

$$S_n = \frac{n}{2}[2 + 2(n - 1)] \quad \boxed{S_n = n^2}$$

EXERCISE 5.3 — SOLUTIONS (Continued)

- 10 Show that a_n forms an AP and find the sum of the first 15 terms.

(i) $a_n = 3 + 4n \rightarrow a_1 = 7, d = 4$ (AP)

$$S_{15} = \frac{15}{2}[14 + 56] = 15 \times 35 \quad \boxed{= 525}$$

(ii) $a_n = 9 - 5n \rightarrow a_1 = 4, d = -5$ (AP)

$$S_{15} = \frac{15}{2}[8 - 70] = 15(-31) \quad \boxed{= -465}$$

- 11 If $S_n = 4n - n^2$, find S_1 , the sum of first two terms, a_2, a_3, a_{10} and a_n .

$$S_1 = 4 - 1 = 3 \quad \boxed{a = 3}$$

$$S_2 = 8 - 4 = 4$$

$$a_2 = S_2 - S_1 = 4 - 3 \quad \boxed{a_2 = 1}$$

$$a_3 = S_3 - S_2 = 3 - 4 = -1$$

$$a_{10} = S_{10} - S_9 = -60 - (-45) = -15$$

$$a_n = S_n - S_{n-1} \quad \boxed{a_n = 5 - 2n}$$



Remember : Remember : $a_n = S_n - S_{n-1}$.

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PART 5 (Ex 5.3)

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EXERCISE 5.3 — SOLUTIONS (Continued)

- 12 Find the sum of the first 40 positive integers divisible by 6.

$$6, 12, 18, \dots, 240 : a = 6, d = 6, n = 40$$

$$S_{40} = \frac{40}{2} [12 + 39(6)]$$

$$S_{40} = 20(12 + 234) = 20 \times 246 = 4920$$

- 13 Find the sum of the first 15 multiples of 8.

$$8, 16, 24, \dots, 120 : a = 8, d = 8, n = 15$$

$$S_{15} = \frac{15}{2} [16 + 14(8)]$$

$$S_{15} = \frac{15}{2} (128) = 15 \times 64 = 960$$

- 14 Find the sum of the odd numbers between 0 and 50.

$$1, 3, 5, \dots, 49 : a = 1, d = 2, n = 25$$

$$S_{25} = \frac{25}{2} (1 + 49) = 25 \times 25 = 625$$

- 15 Penalty : ₹200, ₹250, ₹300, ... each day ₹50 more. Total for 30 days' delay?

$$a = 200, d = 50, n = 30$$

$$S_{30} = \frac{30}{2} [400 + 29(50)]$$

$$S_{30} = 15(400 + 1450) = 15 \times 1850 = ₹ 27750$$

EXERCISE 5.3 — SOLUTIONS (Continued)

- 16 ₹700 for seven cash prizes, each ₹20 less than the preceding. Find each prize.

Let the largest prize be a ; $d = -20$, $n = 7$, $S = 700$.

$$700 = \frac{7}{2} [2a + 6(-20)]$$

$$200 = 2a - 120 \Rightarrow a = 160$$

₹ 160, 140, 120, 100, 80, 60, 40

- 17 Each section of Class I...XII plants trees equal to its class number; 3 sections each. Total trees?

Class k plants $3k$ trees (3 sections).

$$\text{Total} = 3(1 + 2 + \dots + 12)$$

$$= 3 \times \frac{12(13)}{2} = 3 \times 78 = 234 \text{ trees}$$

- 18 Total length of a spiral of 13 semicircles, radii 0.5, 1.0, 1.5, ... cm. ($\pi = \frac{22}{7}$)

Length of each semicircle = πr ; radii form an AP.

$$\text{Length} = \pi(0.5 + 1.0 + \dots \text{ to 13 terms})$$

$$= \frac{22}{7} \times \frac{13}{2} (0.5 + 6.5)$$

$$= \frac{22}{7} \times 45.5 = 143 \text{ cm}$$



Remember :

For word problems, first list the numbers, then read off a , d and n .



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PART 6 (Ex 5.3)

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EXERCISE 5.3 — SOLUTIONS (Continued)

- 19 200 logs are stacked : 20, 19, 18, ... per row. How many rows, and how many logs in the top row?

$$a = 20, d = -1, S_n = 200$$

$$200 = \frac{n}{2} [40 + (n-1)(-1)]$$

$$400 = n(41 - n) \Rightarrow n^2 - 41n + 400 = 0$$

$$(n-16)(n-25) = 0 \Rightarrow n = 16 \text{ or } 25$$

$n = 25$ gives a negative top row, so take $n = 16$.

16 rows

Top row : $a_{16} = 20 + 15(-1) = 5$ **5 logs**

EXERCISE 5.3 — SOLUTIONS (Continued)

- 20 Potato race : bucket 5 m from the first potato, then potatoes 3 m apart, 10 potatoes. Total distance run?

For each potato she runs there and back, i.e. twice the distance.

Distances : 2(5), 2(8), 2(11), ... to 10 terms

Total = 2(5 + 8 + 11 + ... to 10 terms)

$$= 2 \times \frac{10}{2} [2(5) + 9(3)]$$

$$= 2 \times 5(10 + 27) = 2 \times 185 = 370 \text{ m}$$



Remember : Read each situation carefully — set up the AP, then apply the sum formula.



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