



CLASS 10 MATHS

PART Ex 5.4

ARITHMETIC PROGRESSIONS

STUDY • PRACTICE • SCORE HIGH

CONCEPTS USED

Note : Exercise 5.4 is optional and has no separate worked examples in NCERT. It uses : $a_n = a + (n - 1)d$ and $S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + \ell)$.

EXERCISE 5.4 (Optional) — SOLUTIONS

- 1** Which term of the AP : 121, 117, 113, ... is its first negative term?

$$a = 121, d = 117 - 121 = -4$$

$$a_n = 121 + (n - 1)(-4) = 125 - 4n$$

For the first negative term : $a_n < 0$

$$125 - 4n < 0 \Rightarrow 4n > 125 \Rightarrow n > 31.25$$

$$\text{So } n = 32 ; a_{32} = 125 - 128 = -3$$

The 32nd term (= -3) is the first negative term

- 2** The sum of the 3rd and 7th terms of an AP is 6 and their product is 8. Find S_{16} .

$$a_3 + a_7 = 6 \Rightarrow 2a + 8d = 6 \Rightarrow a + 4d = 3 \Rightarrow a = 3 - 4d$$

$$a_3 \cdot a_7 = 8 \Rightarrow (3 - 2d)(3 + 2d) = 8$$

$$9 - 4d^2 = 8 \Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm \frac{1}{2}$$

$$\text{If } d = \frac{1}{2}, a = 1 : S_{16} = 8[2 + 7.5] = 76$$

$$\text{If } d = -\frac{1}{2}, a = 5 : S_{16} = 8[10 - 7.5] = 20$$

EXERCISE 5.4 (Continued)

- 3** A ladder : rungs 25 cm apart, lengths from 45 cm (bottom) to 25 cm (top), top & bottom $2\frac{1}{2}$ m apart. Length of wood for the rungs?

$$\text{Number of rungs} = \frac{250}{25} + 1 = 11$$

The lengths form an AP : $a = 45, \ell = 25, n = 11$

$$\begin{aligned} \text{Wood} &= S_{11} = \frac{11}{2}(45 + 25) \\ &= \frac{11}{2} \times 70 = 385 \text{ cm} \end{aligned}$$

- 4** Houses in a row are numbered 1 to 49. Find x so that the sum of numbers before house x equals the sum after it.

$$S_{x-1} = S_{49} - S_x$$

$$\frac{(x-1)x}{2} = 1225 - \frac{x(x+1)}{2}$$

$$x(x-1) = 2450 - x(x+1)$$

$$2x^2 = 2450 \Rightarrow x^2 = 1225 \quad x = 35$$

- 5** A terrace has 15 steps, each 50 m long, rise $\frac{1}{4}$ m, tread $\frac{1}{2}$ m. Total volume of concrete?

Volume of the k -th step = length $\times$ tread $\times$ height.

$$\text{Volume of } k\text{-th step} = 50 \times \frac{1}{2} \times \frac{k}{4} = 6.25k \text{ m}^3$$

$$\text{Total} = 6.25(1 + 2 + \dots + 15)$$

$$= 6.25 \times 120 = 750 \text{ m}^3$$

**Remember :**

Exercise 5.4 is optional (not for the examination), but it strengthens problem solving. Always translate the situation into a, d and n first.



UNDERSTAND CONCEPTS



PRACTICE REGULARLY



SCORE EXCELLENT



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