✦ EXERCISE 10.2 ✦

1 Compute the magnitude of $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}$, $\vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$.

SOLUTION

- $|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$
- $|\vec{b}| = \sqrt{2^2 + (-7)^2 + (-3)^2} = \sqrt{4 + 49 + 9} = \sqrt{62}$
- $|\vec{c}| = \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1$

2 Write two ***different*** vectors having the ***same magnitude***.

SOLUTION

$\vec{p} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{q} = 2\hat{i} - \hat{j} + \hat{k}$ — both have magnitude $\sqrt{1+1+4} = \sqrt{6}$, yet point in different directions.

3 Write two ***different*** vectors having the ***same direction***.

SOLUTION

$\vec{p} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{q} = 100\hat{i} + 100\hat{j} + 200\hat{k} = 100\,\vec{p}$ — same direction (positive scalar multiple), different magnitudes.

4 Find x and y so that $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are ***equal***.

SOLUTION

Equal vectors have equal components: $x\hat{i} + y\hat{j} = 2\hat{i} + 3\hat{j} \Rightarrow \boxed{x = 2,\ y = 3}$

EXERCISE 10.2 (CONTD.)

5 *Find the scalar and vector components of the vector with initial point* $(2,1)$ *and terminal point* $(-5,7)$.

SOLUTION

- $\overrightarrow{AB} = (\text{terminal}) - (\text{initial}) = (-5-2)\hat{i} + (7-1)\hat{j} = -7\hat{i} + 6\hat{j}$
- ***Vector components:*** $-7\hat{i},\ 6\hat{j}$; ***Scalar components:*** $-7,\ 6$.

6 *Find the sum of* $\vec{a} = \hat{i} - 2\hat{j} + \hat{k},\ \vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k},\ \vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

SOLUTION

$$\vec{a} + \vec{b} + \vec{c} = (1-2+1)\hat{i} + (-2+4-6)\hat{j} + (1+5-7)\hat{k} = \boxed{-4\hat{j} - \hat{k}}$$

7 *Find the unit vector in the direction of* $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

SOLUTION

$$|\vec{a}| = \sqrt{1+1+4} = \sqrt{6} \Rightarrow \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

8 *Find the unit vector in the direction of* $\overrightarrow{PQ}$, *where* $P(1,2,3)$ *and* $Q(4,5,6)$.

SOLUTION

- $\overrightarrow{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$
- $|\overrightarrow{PQ}| = \sqrt{9+9+9} = 3\sqrt{3} \Rightarrow \widehat{PQ} = \dfrac{3\hat{i} + 3\hat{j} + 3\hat{k}}{3\sqrt{3}} = \dfrac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

✦ EXERCISE 10.2 (CONTD.) ✦

9 *For* $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ *and* $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$*, find the unit vector in the direction of* $\vec{a} + \vec{b}$*.*

SOLUTION

$\vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k} = \hat{i} + \hat{k}$, $|\vec{a} + \vec{b}| = \sqrt{2} \Rightarrow$ *unit* $= \dfrac{1}{\sqrt{2}}(\hat{i} + \hat{k})$

10 *Find the vector in the direction of* $5\hat{i} - \hat{j} + 2\hat{k}$ *which has magnitude* 8 *units.*

SOLUTION

$|\vec{a}| = \sqrt{25+1+4} = \sqrt{30} \Rightarrow$ *required* $= 8\,\hat{a} = \dfrac{8}{\sqrt{30}}(5\hat{i} - \hat{j} + 2\hat{k})$

11 *Show that the vectors* $2\hat{i} - 3\hat{j} + 4\hat{k}$ *and* $-4\hat{i} + 6\hat{j} - 8\hat{k}$ *are collinear.*

SOLUTION

$-4\hat{i} + 6\hat{j} - 8\hat{k} = -2(2\hat{i} - 3\hat{j} + 4\hat{k})$. *Ratios* $\dfrac{2}{-4} = \dfrac{-3}{6} = \dfrac{4}{-8} = -\dfrac{1}{2}$. *Since one is a scalar multiple of the other, they are* ***collinear****.*

12 *Find the direction cosines of the vector* $\hat{i} + 2\hat{j} + 3\hat{k}$*.*

SOLUTION

$|\vec{r}| = \sqrt{1+4+9} = \sqrt{14} \Rightarrow$ *d.c.'s* $= \left(\dfrac{1}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}\right)$

13 *Find the direction cosines of the vector joining* $A(1, 2, -3)$ *and* $B(-1, -2, 1)$*, directed from* A *to* B*.*

SOLUTION

- $\overrightarrow{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + (1+3)\hat{k} = -2\hat{i} - 4\hat{j} + 4\hat{k}$
- $|\overrightarrow{AB}| = \sqrt{4+16+16} = 6 \Rightarrow$ *d.c.'s* $= \left(-\dfrac{1}{3}, -\dfrac{2}{3}, \dfrac{2}{3}\right)$

EXERCISE 10.2 (CONTD.)

14 *Show that the vector $\hat{i}+\hat{j}+\hat{k}$ is equally inclined to the axes OX, OY, OZ.*

SOLUTION

$|\vec{r}| = \sqrt{1+1+1} = \sqrt{3}$. Direction cosines $\cos\alpha = \cos\beta = \cos\gamma = \dfrac{1}{\sqrt{3}}$. Since all three are equal, $\alpha = \beta = \gamma$ — the vector is **equally inclined** to all three axes.

15 *Find the position vector of the point R dividing the join of $P(\hat{i}+2\hat{j}-\hat{k})$ and $Q(-\hat{i}+\hat{j}+\hat{k})$ in the ratio $2:1$ (i) internally (ii) externally.*

SOLUTION

- **(i) Internal:** $\vec{r} = \dfrac{2\vec{q}+1\vec{p}}{2+1} = \dfrac{2(-\hat{i}+\hat{j}+\hat{k})+(\hat{i}+2\hat{j}-\hat{k})}{3} = \boxed{-\frac{1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}}$
- **(ii) External:** $\vec{r} = \dfrac{2\vec{q}-1\vec{p}}{2-1} = 2(-\hat{i}+\hat{j}+\hat{k})-(\hat{i}+2\hat{j}-\hat{k}) = \boxed{-3\hat{i}+3\hat{k}}$

16 *Find the position vector of the mid-point of the vector joining $P(2,3,4)$ and $Q(4,1,-2)$.*

SOLUTION

$$\vec{r} = \frac{\vec{p}+\vec{q}}{2} = \frac{(2\hat{i}+3\hat{j}+4\hat{k})+(4\hat{i}+\hat{j}-2\hat{k})}{2} = \boxed{3\hat{i}+2\hat{j}+\hat{k}}$$

17 *Show that the points with position vectors $\vec{a}=3\hat{i}-4\hat{j}-4\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$ form a **right-angled triangle**.*

SOLUTION

- $\overrightarrow{BA} = \vec{a}-\vec{b} = \hat{i}-3\hat{j}-5\hat{k}$, $|\overrightarrow{BA}|^2 = 35$
- $\overrightarrow{AC} = \vec{c}-\vec{a} = -2\hat{i}+\hat{j}-\hat{k}$, $|\overrightarrow{AC}|^2 = 6$
- $\overrightarrow{CB} = \vec{b}-\vec{c} = \hat{i}+2\hat{j}+6\hat{k}$, $|\overrightarrow{CB}|^2 = 41$
- Since $35+6=41$, i.e. $|\overrightarrow{BA}|^2+|\overrightarrow{AC}|^2 = |\overrightarrow{CB}|^2$, by the converse of Pythagoras the triangle is **right-angled at A**.