SOME APPLICATIONS OF TRIGONOMETRY

CLASS - 10

1 INTRODUCTION

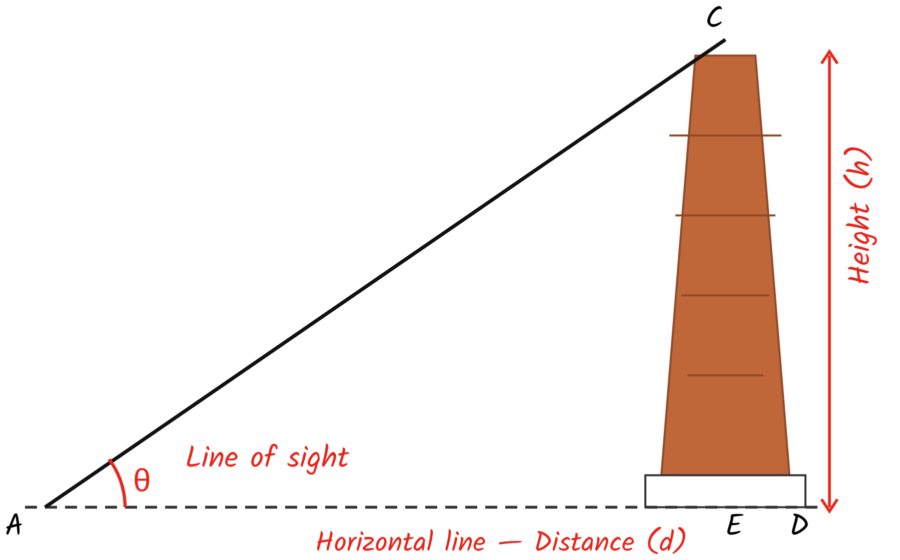
Trigonometry helps us find :

- Heights of buildings, towers, trees
- Distances between objects
- Width of rivers
- Heights of mountains
- Distance of ships from lighthouses

...without directly measuring them.

This chapter mainly uses :

$\sin\theta$, $\cos\theta$, $\tan\theta$



In right $\triangle ACD$, if we know (i) Distance d (ii) Height of observer (if any) (iii) Angle θ — then we can find the height h.

2 IMPORTANT TERMS

(A) LINE OF SIGHT

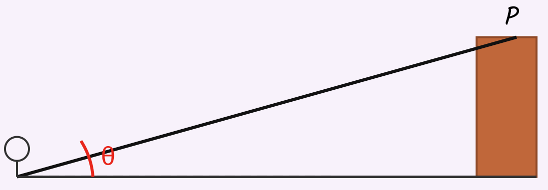
The line drawn from the observer's eye to the object.

Example: looking at the top of a tower, kite, aeroplane etc.

The imaginary line joining the eye and the object is called the ***Line of Sight***.

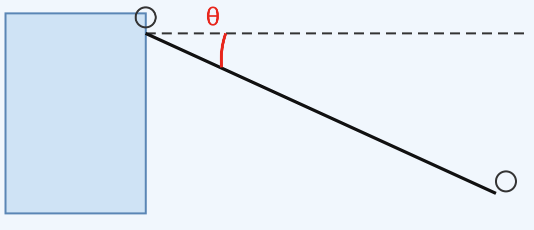
(B) ANGLE OF ELEVATION

When an observer looks **upward** at an object, the angle between the horizontal line and the line of sight is called the ***Angle of Elevation***.



(C) ANGLE OF DEPRESSION

When an observer looks **downward** at an object, the angle between the horizontal line and the line of sight is called the ***Angle of Depression***.



Examples

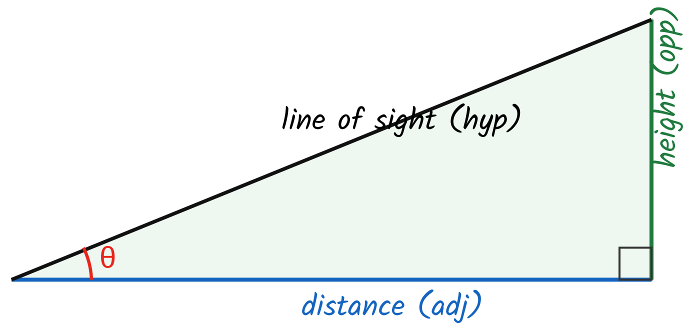
Angle of elevation: top of a building, top of a mountain, kite in the sky.

Angle of depression: looking down from a balcony, from a lighthouse, from a bridge.

SOME APPLICATIONS OF TRIGONOMETRY

3 THE RIGHT-TRIANGLE TOOL

- *Every height & distance problem becomes a **right triangle**.*
- *The **vertical** side = height, the **horizontal** side = distance, the slanting side = line of sight.*



$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\text{height}}{\text{distance}}$$

***Key idea:** the **angle of depression** equals the **angle of elevation** from the other point (alternate angles, since the horizontal lines are parallel).*

4 STANDARD VALUES YOU MUST KNOW

θ	30°	45°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Steps to solve any problem:

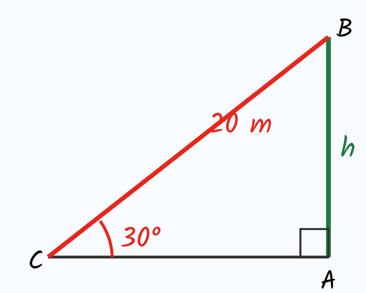
- *Draw a neat figure; mark the right angle, known sides & the angle.*
- *Pick the ratio that links the **known** and the **unknown** side.*
- *Put in the standard value and solve. Write the unit.*

SOME APPLICATIONS OF TRIGONOMETRY

9.1 EXERCISE 9.1 — SOLUTIONS

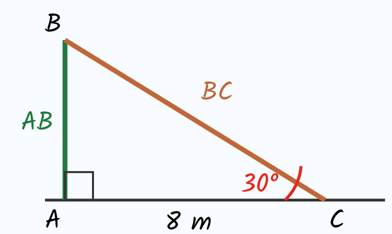
1 *A circus artist climbs a* ***20 m*** *long rope tied from the top of a vertical pole to the ground. The rope makes* ***30°*** *with the ground. Find the height of the pole.*

- **Given:** rope (hyp) $AC = 20$ m, angle $= 30^\circ$, pole $AB = h$.
- $\sin 30^\circ = \dfrac{AB}{AC} = \dfrac{h}{20}$
- $\dfrac{1}{2} = \dfrac{h}{20} \Rightarrow h = 20 \times \dfrac{1}{2}$
- $\boxed{\text{Height of pole} = 10 \text{ m}}$



2 *A tree breaks in a storm; the top touches the ground making* ***30°*** *with it. The foot of the tree to the point where the top touches is* ***8 m****. Find the height of the tree.*

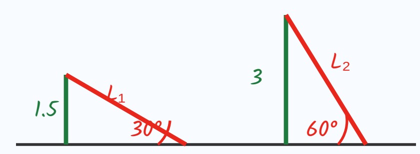
- **Given:** $AC = 8$ m, broken part BC makes 30°. Standing part $= AB$.
- $\tan 30^\circ = \dfrac{AB}{AC} \Rightarrow AB = 8\tan 30^\circ = \dfrac{8}{\sqrt{3}}$ m.
- $\cos 30^\circ = \dfrac{AC}{BC} \Rightarrow BC = \dfrac{8}{\cos 30^\circ} = \dfrac{16}{\sqrt{3}}$ m.
- Height $= AB + BC = \dfrac{8}{\sqrt{3}} + \dfrac{16}{\sqrt{3}} = \dfrac{24}{\sqrt{3}}$
- $\boxed{\text{Height of tree} = 8\sqrt{3} \approx 13.86 \text{ m}}$



SOME APPLICATIONS OF TRIGONOMETRY

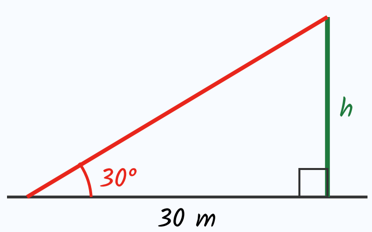
3 *Two slides: for younger children, top at height* ***1.5 m****, inclined at* ***30°****; for elder children, top at* ***3 m****, inclined at* ***60°****. Find the length of each slide.*

- ***Slide 1:*** $\sin 30^\circ = \dfrac{1.5}{L_1} \Rightarrow L_1 = \dfrac{1.5}{\sin 30^\circ} = 1.5 \times 2.$ $\boxed{L_1 = 3\ m}$
- ***Slide 2:*** $\sin 60^\circ = \dfrac{3}{L_2} \Rightarrow L_2 = \dfrac{3}{\sin 60^\circ} = \dfrac{3 \times 2}{\sqrt{3}}.$ $\boxed{L_2 = 2\sqrt{3} \approx 3.46\ m}$



4 *The angle of elevation of the top of a tower from a point* ***30 m*** *away from its foot is* ***30°****. Find the height of the tower.*

- ***Given:*** *distance* $= 30$ *m, angle* $= 30^\circ$*, height* $= h$.
- $\tan 30^\circ = \dfrac{h}{30} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{30}$
- $h = \dfrac{30}{\sqrt{3}} = 10\sqrt{3}$
- $\boxed{\textit{Height of tower} = 10\sqrt{3} \approx 17.32\ m}$



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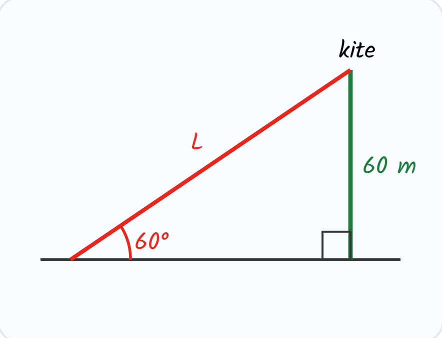
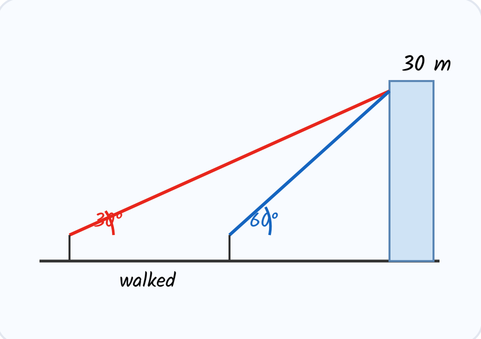
5 *A kite is flying at a height of **60 m**. The string is tied to a point on the ground and inclined at **60°**. Find the length of the string (no slack).*

- **Given:** height $= 60$ m, angle $= 60^\circ$, string (hyp) $= L$.
- $\sin 60^\circ = \dfrac{60}{L} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{60}{L}$
- $L = \dfrac{60 \times 2}{\sqrt{3}} = \dfrac{120}{\sqrt{3}} = 40\sqrt{3}$
- **Length of string** $= 40\sqrt{3} \approx 69.28$ **m**



6 *A **1.5 m** tall boy stands away from a **30 m** tall building. The angle of elevation to the top rises from **30°** to **60°** as he walks towards it. Find the distance he walked.*

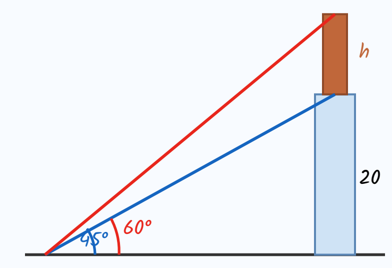
- Effective height above eye $= 30 - 1.5 = 28.5$ m.
- At 30°: $x_1 = \dfrac{28.5}{\tan 30^\circ} = 28.5\sqrt{3}$. At 60°: $x_2 = \dfrac{28.5}{\tan 60^\circ} = \dfrac{28.5}{\sqrt{3}}$.
- Distance walked $= x_1 - x_2 = 28.5\left(\sqrt{3} - \dfrac{1}{\sqrt{3}}\right) = 28.5 \times \dfrac{2}{\sqrt{3}}$
- **Distance walked** $= 19\sqrt{3} \approx 32.9$ **m**



SOME APPLICATIONS OF TRIGONOMETRY

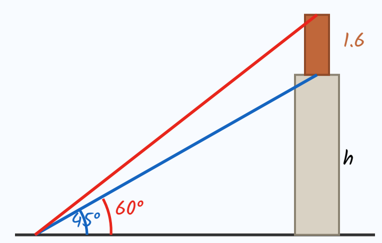
7 *From a point on the ground, the angles of elevation of the* ***bottom*** *and the* ***top*** *of a transmission tower on a* ***20 m*** *building are* ***45°*** *and* ***60°****. Find the height of the tower.*

- *Let distance from point to building* $= x$. *Bottom of tower (top of building, 20 m) at* 45°:
- $\tan 45^\circ = \dfrac{20}{x} \Rightarrow x = 20$ *m.*
- *Top of tower at* 60°: $\tan 60^\circ = \dfrac{20+h}{x} \Rightarrow \sqrt{3} = \dfrac{20+h}{20} \Rightarrow 20 + h = 20\sqrt{3}$.
- ***Height of tower*** $= 20(\sqrt{3}-1) \approx 14.64$ ***m***



8 *A statue* ***1.6 m*** *tall stands on a pedestal. From a point on the ground, the elevation of the top of the statue is* ***60°*** *and of the top of the pedestal is* ***45°****. Find the height of the pedestal.*

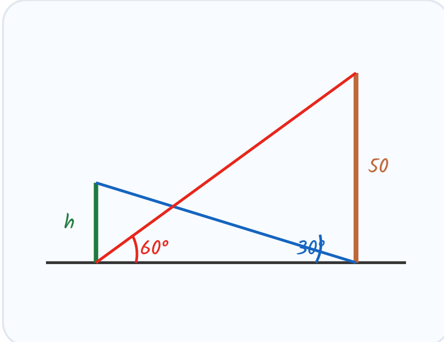
- *Let pedestal height* $= h$, *distance* $= d$. *Top of pedestal at* 45°: $\tan 45^\circ = \dfrac{h}{d} \Rightarrow d = h$.
- *Top of statue at* 60°: $\tan 60^\circ = \dfrac{h+1.6}{d} = \dfrac{h+1.6}{h} = \sqrt{3}$.
- $h + 1.6 = \sqrt{3}\,h \Rightarrow 1.6 = h(\sqrt{3}-1) \Rightarrow h = \dfrac{1.6}{\sqrt{3}-1} = 0.8(\sqrt{3}+1)$.
- ***Height of pedestal*** $= 0.8(\sqrt{3}+1) \approx 2.19$ ***m***



SOME APPLICATIONS OF TRIGONOMETRY

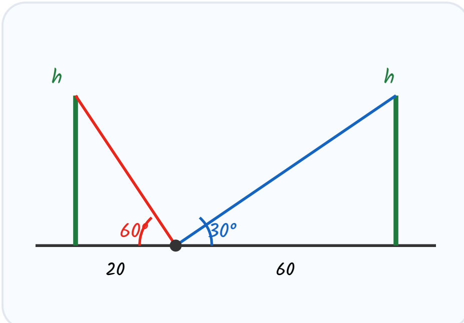
9 *The elevation of the top of a building from the foot of a tower is **30°**, and the elevation of the top of the tower from the foot of the building is **60°**. If the tower is **50 m** high, find the height of the building.*

- *Let distance between feet* $= d$, *building height* $= h$.
- *Tower from foot of building:* $\tan 60^\circ = \dfrac{50}{d} \Rightarrow d = \dfrac{50}{\sqrt{3}}$.
- *Building from foot of tower:* $\tan 30^\circ = \dfrac{h}{d} \Rightarrow h = d\tan 30^\circ = \dfrac{50}{\sqrt{3}} \cdot \dfrac{1}{\sqrt{3}} = \dfrac{50}{3}$.
- ***Height of building*** $= \dfrac{50}{3} \approx 16.67$ ***m***



10 *Two poles of equal height stand on either side of an **80 m** wide road. From a point between them, the elevations of their tops are **60°** and **30°**. Find the height of the poles and the distances of the point from them.*

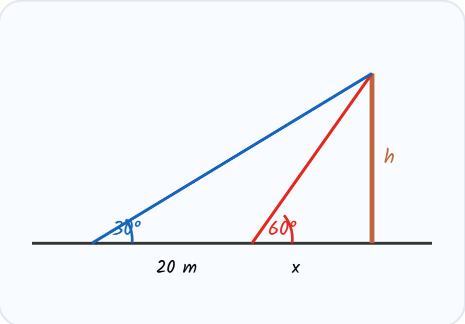
- *Let height* $= h$, *the point be* x *m from the first pole, so* $(80 - x)$ *from the other.*
- $\tan 60^\circ = \dfrac{h}{x} \Rightarrow h = x\sqrt{3}$; $\quad \tan 30^\circ = \dfrac{h}{80 - x} \Rightarrow h = \dfrac{80 - x}{\sqrt{3}}$.
- $x\sqrt{3} = \dfrac{80 - x}{\sqrt{3}} \Rightarrow 3x = 80 - x \Rightarrow x = 20$. *So* $h = 20\sqrt{3}$.
- ***Height*** $= 20\sqrt{3} \approx 34.64$ ***m; distances*** $= 20$ ***m and*** 60 ***m***



SOME APPLICATIONS OF TRIGONOMETRY

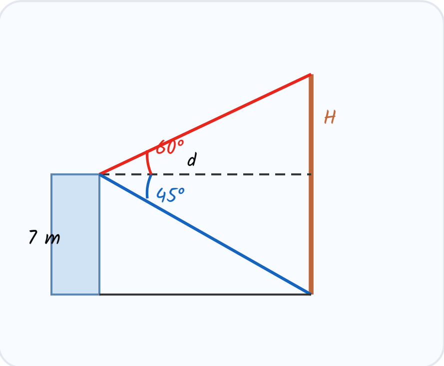
11 *A TV tower stands on a bank of a canal. From a point on the opposite bank directly opposite the tower, the elevation of the top is **60°**. From a point **20 m** back (in line), the elevation is **30°**. Find the height of the tower and the width of the canal.*

- *Let width* $= x$, *height* $= h$. *At the near point:* $\tan 60^\circ = \dfrac{h}{x} \Rightarrow h = x\sqrt{3}$.
- *At the far point (20 m back):* $\tan 30^\circ = \dfrac{h}{x+20} \Rightarrow h = \dfrac{x+20}{\sqrt{3}}$.
- $x\sqrt{3} = \dfrac{x+20}{\sqrt{3}} \Rightarrow 3x = x + 20 \Rightarrow x = 10$. *So* $h = 10\sqrt{3}$.
- ***Height*** $= 10\sqrt{3} \approx 17.32$ ***m; width of canal*** $= 10$ ***m***



12 *From the top of a **7 m** building, the angle of elevation of the top of a cable tower is **60°** and the angle of depression of its foot is **45°**. Find the height of the tower.*

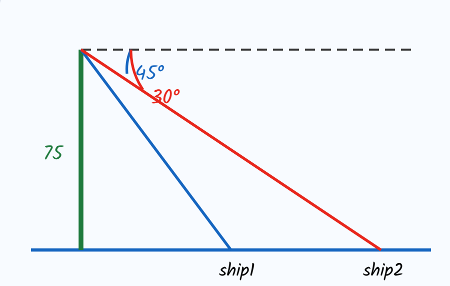
- *Let horizontal distance between building & tower* $= d$.
- *Depression of foot* $= 45^\circ$: $\tan 45^\circ = \dfrac{7}{d} \Rightarrow d = 7$ *m*.
- *Elevation of top* $= 60^\circ$: *height above building* $= H$, $\tan 60^\circ = \dfrac{H}{d} \Rightarrow H = 7\sqrt{3}$.
- *Total height* $= 7 + 7\sqrt{3}$.
- ***Height of tower*** $= 7(1+\sqrt{3}) \approx 19.12$ ***m***



SOME APPLICATIONS OF TRIGONOMETRY

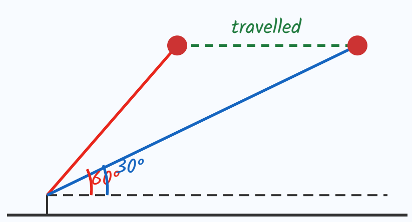
13 *From the top of a **75 m** lighthouse, the angles of depression of two ships (one behind the other, same side) are **30°** and **45°**. Find the distance between the two ships.*

- Depression = elevation from ship. Nearer ship at 45°: $\tan 45^\circ = \dfrac{75}{d_1} \Rightarrow d_1 = 75$ m.
- Farther ship at 30°: $\tan 30^\circ = \dfrac{75}{d_2} \Rightarrow d_2 = 75\sqrt{3}$ m.
- Distance between ships $= d_2 - d_1 = 75\sqrt{3} - 75$.
- $\boxed{\text{Distance} = 75(\sqrt{3} - 1) \approx 54.9 \text{ m}}$



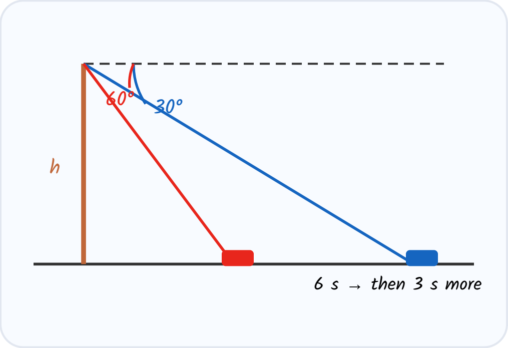
14 *A **1.2 m** tall girl sees a balloon at height **88.2 m**. The elevation of the balloon from her eyes is **60°**; after a while it reduces to **30°**. Find the distance travelled by the balloon.*

- Height above the girl's eyes $= 88.2 - 1.2 = 87$ m.
- At 60°: $x_1 = \dfrac{87}{\tan 60^\circ} = \dfrac{87}{\sqrt{3}} = 29\sqrt{3}$. At 30°: $x_2 = \dfrac{87}{\tan 30^\circ} = 87\sqrt{3}$.
- Distance travelled $= x_2 - x_1 = 87\sqrt{3} - 29\sqrt{3}$.
- $\boxed{\text{Distance} = 58\sqrt{3} \approx 100.46 \text{ m}}$



SOME APPLICATIONS OF TRIGONOMETRY

15 *A straight highway leads to the foot of a tower. From the top of the tower a car is seen at an angle of depression of **30°**; it approaches at uniform speed. Six seconds later the depression is **60°**. Find the time taken by the car to reach the foot of the tower from this point.*



- Let tower height $= h$. At 30°: distance $= \dfrac{h}{\tan 30^\circ} = h\sqrt{3}$. At 60°: distance $= \dfrac{h}{\tan 60^\circ} = \dfrac{h}{\sqrt{3}}$.
- Distance covered in 6 $s = h\sqrt{3} - \dfrac{h}{\sqrt{3}} = h \cdot \dfrac{2}{\sqrt{3}}$. Remaining distance $= \dfrac{h}{\sqrt{3}}$.
- Speed $= \dfrac{2h/\sqrt{3}}{6}$. Time $= \dfrac{\text{remaining}}{\text{speed}} = \dfrac{h/\sqrt{3}}{2h/(6\sqrt{3})} = \dfrac{6}{2}$.
- **Time to reach the foot $= 3$ seconds**

Answer key (Exercise 9.1): 1) 10 m 2) $8\sqrt{3}$ m 3) 3 m, $2\sqrt{3}$ m 4) $10\sqrt{3}$ m 5) $40\sqrt{3}$ m 6) $19\sqrt{3}$ m 7) $20(\sqrt{3}-1)$ m 8) $0.8(\sqrt{3}+1)$ m 9) $\frac{50}{3}$ m 10) $20\sqrt{3}$ m; 20 m, 60 m 11) $10\sqrt{3}$ m; 10 m 12) $7(1+\sqrt{3})$ m 13) $75(\sqrt{3}-1)$ m 14) $58\sqrt{3}$ m 15) 3 s.