

**MID-TERM EXAMINATION — Class 10 Mathematics (Set 3)****Chapters:** Real Numbers, Polynomials, Linear Equations, Triangles, Coordinate Geometry, Circles, Areas Related to Circles, Trigonometry & its Applications**Max Marks:**
80**Name:** _____**Time: 2 Hours****General Instructions:** All questions are compulsory. Five sections — A (20×1), B (5×2), C (6×3), D (4×5), E (3×4, case-studies). Draw neat figures where required. Take $\pi = \frac{22}{7}$ unless stated otherwise.**SECTION A**

Multiple Choice / Assertion-Reason — 1 mark each

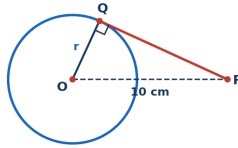
- Q1.** The HCF of two consecutive natural numbers is: [1]
- (a) 0
(b) 1
(c) 2
(d) the smaller number
- Q2.** For two positive integers a and b , $\text{HCF}(a, b) \times \text{LCM}(a, b)$ equals: [1]
- (a) $a + b$
(b) $a - b$
(c) $a \times b$
(d) $\frac{a}{b}$
- Q3.** The zeroes of the polynomial $x^2 - 3$ are: [1]
- (a) ± 3
(b) $\pm\sqrt{3}$
(c) 3, 0
(d) $\sqrt{3}, 0$
- Q4.** A quadratic polynomial whose zeroes are -3 and 4 is: [1]
- (a) $x^2 - x - 12$
(b) $x^2 + x - 12$
(c) $x^2 - x + 12$
(d) $x^2 + 7x + 12$
- Q5.** The equations $kx - 2y = 3$ and $3x + y = 5$ have a unique solution for all values of k except: [1]
- (a) $k = 6$
(b) $k = -6$
(c) $k = 2$
(d) $k = 3$
- Q6.** The distance of the point $P(3, 4)$ from the x -axis is: [1]
- (a) 3
(b) 4
(c) 5
(d) 7
- Q7.** The distance between the points $(a, 0)$ and $(0, b)$ is: [1]
- (a) $a + b$
(b) $\sqrt{a^2 + b^2}$
(c) ab
(d) $a - b$
- Q8.** If the ratio of the perimeters of two similar triangles is $5 : 3$, the ratio of their areas is: [1]

- (a) 5 : 3
- (b) 25 : 9
- (c) 3 : 5
- (d) $\sqrt{5} : \sqrt{3}$

Q9. The number of tangents that can be drawn to a circle from a point lying on the circle is: [1]

- (a) 0
- (b) 1
- (c) 2
- (d) infinite

Q10. In the figure, PQ is a tangent from P where $OP = 10$ cm. If $PQ = 8$ cm, the radius $r =$ [1]



- (a) 4 cm
- (b) 6 cm
- (c) $\sqrt{164}$ cm
- (d) 12 cm

Q11. The value of $\frac{\sin 60^\circ}{\cos 30^\circ}$ is: [1]

- (a) 0
- (b) 1
- (c) $\sqrt{3}$
- (d) $\frac{1}{2}$

Q12. The value of $\cot 45^\circ$ is: [1]

- (a) 0
- (b) 1
- (c) $\sqrt{3}$
- (d) $\frac{1}{\sqrt{3}}$

Q13. $7 \times 11 \times 13 + 13$ is a: [1]

- (a) prime number
- (b) composite number
- (c) an odd prime
- (d) none

Q14. The angle of elevation of the sun is 45° when the length of the shadow of a pole equals: [1]

- (a) twice its height
- (b) its height
- (c) $\sqrt{3}$ times its height
- (d) half its height

Q15. The area of a circle whose diameter is 14 cm is: [1]

- (a) 154 cm^2
- (b) 44 cm^2
- (c) 616 cm^2
- (d) 22 cm^2

Q16. If an arc of length 22 cm subtends the centre of a circle of radius 21 cm, the angle of the sector is: [1]

- (a) 30°
- (b) 45°

(c) 60°

(d) 90°

Q17. The sum of the zeroes of $3x^2 - 5x + 2$ is:

[1]

(a) $\frac{5}{3}$

(b) $\frac{2}{3}$

(c) $-\frac{5}{3}$

(d) 5

Q18. The distance between two parallel tangents of a circle of radius 7 cm is:

[1]

(a) 7 cm

(b) 14 cm

(c) 3.5 cm

(d) 49 cm

Q19. **Assertion (A):** For any two positive integers, $\text{HCF} \times \text{LCM} = \text{product of the numbers}$.

[1]

Reason (R): This relationship holds for exactly two positive integers.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

Q20. **Assertion (A):** The degree of a quadratic polynomial is 2.

[1]

Reason (R): A polynomial of degree 2 has at most two zeroes.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

SECTION B

Very Short Answer — 2 marks each

Q21. Find the HCF and LCM of 12, 15 and 21.

[2]

Q22. If α, β are the zeroes of $x^2 - 7x + 10$, find the value of $\alpha^2 + \beta^2$.

[2]

Q23. Solve for x and y : $x + 2y = 6$ and $3x - 4y = 8$.

[2]

Q24. The ratio of the corresponding sides of two similar triangles is 4 : 9. If the area of the smaller triangle is 48 cm^2 , find the area of the larger triangle.

[2]

Q25. Prove that $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \tan^2 A$.

[2]

SECTION C

Short Answer — 3 marks each

Q26. Prove that $3 + 2\sqrt{5}$ is an irrational number.

[3]

Q27. Find the zeroes of $\sqrt{3}x^2 + 10x + 7\sqrt{3}$ and verify the relationship between the zeroes and the coefficients.

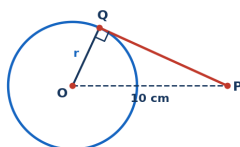
[3]

Q28. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from its numerator, and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

[3]

Q29. Prove that the lengths of tangents drawn from an external point to a circle are equal.

[3]



Q30. Find the area of the quadrilateral whose vertices, in order, are $(-4, -2)$, $(-3, -5)$, $(3, -2)$ and $(2, 3)$.

[3]

- Q31.** Prove that $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \csc \theta$. [3]

SECTION D

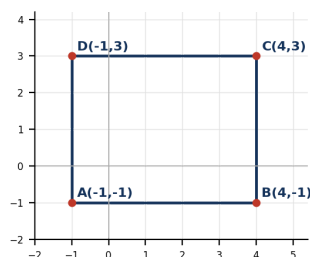
Long Answer — 5 marks each

- Q32. Prove:** The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides. [5]
- Q33.** A train travels **300** km at a uniform speed. If the speed had been **5** km/h more, it would have taken **2** hours less for the journey. Find the speed of the train. [5]
- Q34.** From a point on the ground, the angle of elevation of the top of a **10** m tall building is 30° . A flag is hoisted at the top of the building and the angle of elevation of the top of the flag-staff from the same point is 45° . Find the length of the flag-staff. (Use $\sqrt{3} = 1.73$) [5]
- Q35.** In a square of side **14** cm, four quadrants each of radius **7** cm are cut off, one at each corner. Find the area of the remaining (shaded) region. [5]

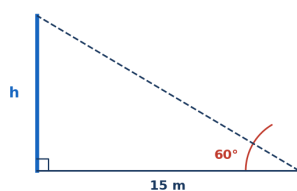
SECTION E

Case-Study Based — 4 marks each

- Q36. Plot of land.** A rectangular plot has corners at $A(-1, -1)$, $B(4, -1)$, $C(4, 3)$ and $D(-1, 3)$ on a coordinate grid (units in metres). [4]



- (i) Find the length AB . [1]
- (ii) Find the length BC . [1]
- (iii) Find the length of the diagonal AC and state the shape of $ABCD$. [2]
- Q37. Observation tower.** A student standing **15** m from the base of a tower finds the angle of elevation of its top to be 60° . [4]



- (i) Find the height of the tower. [2]
- (ii) If the student walks back so that the angle becomes 30° , find the new distance from the base. [1]
- (iii) How far did the student walk back? [1]
- Q38. Circular fountain.** A circular fountain has radius **14** m. A decorative quarter-circle (90°) tiled path runs along one sector. [4]
- (i) Find the area of the whole fountain. [1]
- (ii) Find the area of the quarter-circle sector. [1]
- (iii) Find the length of the arc of that quarter. [2]

— All the Best! —

ANSWER KEY & MARKING SCHEME

SECTION A

Answers

- Q1. (b) 1
- Q2. (c) $a \times b$
- Q3. (b) $\pm\sqrt{3}$
- Q4. (a) $x^2 - x - 12$
- Q5. (b) $k = -6$
- Q6. (b) 4
- Q7. (b) $\sqrt{a^2 + b^2}$
- Q8. (b) $25 : 9$
- Q9. (b) 1
- Q10. (b) 6 cm [$r = \sqrt{10^2 - 8^2}$]
- Q11. (b) 1
- Q12. (b) 1
- Q13. (b) composite number
- Q14. (b) its height
- Q15. (a) 154 cm^2
- Q16. (c) 60°
- Q17. (a) $\frac{5}{3}$
- Q18. (b) 14 cm

SECTION B

Answers

- Q19. $12 = 2^2 \cdot 3$, $15 = 3 \cdot 5$, $21 = 3 \cdot 7$. HCF = 3, LCM = $2^2 \cdot 3 \cdot 5 \cdot 7 = 420$.
- Q20. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 49 - 20 = 29$.
- Q21. From $x = 6 - 2y$: $3(6 - 2y) - 4y = 8 \Rightarrow 18 - 10y = 8 \Rightarrow y = 1$, $x = 4$.
- Q22. Ratio of areas = $4^2 : 9^2 = 16 : 81$. Larger area = $48 \times \frac{16}{81} = 243 \text{ cm}^2$.
- Q23. $\frac{1}{\tan A} + \tan A = \frac{1}{\sec A} + \tan A = \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \frac{1 + \sin A}{\cos A}$. Proved.

SECTION C

Answers

- Q24. If $3 + 2\sqrt{5}$ were rational = r , then $\sqrt{5} = \frac{r-3}{2}$ would be rational — contradiction, as $\sqrt{5}$ is irrational. Hence $3 + 2\sqrt{5}$ is irrational.
- Q25. $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$: discriminant = $100 - 84 = 16$; $x = \frac{-10 \pm 4\sqrt{3}}{2\sqrt{3}}$, zeroes = $-\sqrt{3}$, $-\frac{7\sqrt{3}}{3}$. Sum = $-\frac{10\sqrt{3}}{2\sqrt{3}} = -5$; product = $7 = \frac{7}{1}$. Verified.
- Q26. $\frac{dx}{dy} - 1y = \frac{dx}{dy}13$ and $\frac{dy}{dx}xy + 8 = \frac{dy}{dx}14$. From these $y = 3x - 3$ and $4x = y + 8 \Rightarrow 4x = 3x + 5 \Rightarrow x = 5$, $y = 12$. Fraction = $\frac{5}{12}$.
- Q27. Let PA, PB be tangents from P , centre O . In $\triangle OAP, \triangle OBP$: $OA = OB$ (radii), $\angle OAP = \angle OBP = 90^\circ$, OP common $\Rightarrow$ RHS congruence $\Rightarrow PA = PB$.
- Q28. By the shoelace formula the area = $\frac{1}{2}|14 + 21 + 13 + 8| = \frac{1}{2}(56) = 28 \text{ sq. units}$.
- Q29. Taking LCM and using $\tan\theta \cot\theta = 1$, LHS simplifies to $\frac{\tan^2\theta}{\tan\theta} - \cot^2\theta \tan\theta - \cot\theta + 1 = (\tan\theta + \cot\theta) + 1 = 1 + \sec^2\theta$. Proved.

SECTION D

Answers

- Q30. For similar $\triangle ABC \sim \triangle PQR$ with altitudes AM, PN : $\frac{AM}{PN} = \frac{BC}{QR} = \frac{AB}{PQ} = \frac{AC}{PR}$. Since $\frac{AM}{PN} = \frac{BC}{QR}$, the ratio = $\left(\frac{BC}{QR}\right)^2$.
- Q31. $\frac{300x}{x} - \frac{300x}{x} + 5 = 2 \Rightarrow 1500 = 2x(x + 5) \Rightarrow x^2 + 5x - 750 = 0 \Rightarrow (x - 25)(x + 30) = 0 \Rightarrow x = 25 \text{ km/h}$.

Q32. $\tan 30^\circ = \frac{10}{h} \Rightarrow h = 10 \sqrt{3}$. Top of flag:
 $\tan 45^\circ = \frac{10 + h}{10} \Rightarrow 10 + h = 10 \sqrt{3} \Rightarrow h = 10(\sqrt{3} - 1) = 7.3$ m.

Q33. Four quadrants of radius 7 = one full circle = $\frac{22}{7} \times 49 = 154$ cm². Shaded area = $14^2 - 154 = 196 - 154 = 42$ cm².

SECTION E

Answers

Q34. (i) $AB = 5$ m.
(ii) $BC = 4$ m.
(iii) $AC = \sqrt{5^2 + 4^2} = \sqrt{41}$ m; since opposite sides are equal with right angles, $ABCD$ is a rectangle.

Q35. (i) Height = $15 \tan 60^\circ = 15 \sqrt{3} \approx 25.98$ m.
(ii) New distance = $\frac{15 \sqrt{3}}{\tan 30^\circ} = 45$ m.
(iii) Walked back = $45 - 15 = 30$ m.

Q36. (i) Area = $\frac{22}{7} \times 14^2 = 616$ m².
(ii) Quarter sector = $\frac{616}{4} = 154$ m².
(iii) Arc = $\frac{14 \times 2 \times \frac{22}{7} \times 14}{4} = 22$ m.