

**MID-TERM EXAMINATION — Class 10 Mathematics (Set 2)**

Chapters: GST, Banking, Linear Inequations, Quadratic Equations, Ratio & Proportion, Factorisation, Matrices, Arithmetic Progression, Geometric Progression, Reflection, Mid-point & Section Formula, Equation of a Line, Similarity, Circles, Trigonometric Identities, Probability

Max Marks: 80

Name: _____

Time: 2½ Hours

General Instructions: Answers to this paper must be written on the paper provided separately. You will **not** be allowed to write during the first 15 minutes (reading time). Attempt **all** questions from Section A and **any four** questions from Section B. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks. Mathematical tables and squared paper are provided.

SECTION A**40 Marks — Attempt ALL questions***Attempt all questions from this Section.*

- Q1.** (i) A dealer in Delhi buys goods worth Rs. 8000 from a manufacturer in Delhi and sells them to a consumer in Delhi at a profit of Rs. 2000. If all transactions are intra-state and the GST rate is 12%, find the GST paid by the dealer to the government on his value addition. [3]
- (ii) Priya deposits Rs. 800 per month in a recurring deposit account for 18 months at 10% per annum. Find the interest earned and the maturity value. [3]
- (iii) If $A = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$, find $A^2 - 2A + I$, where I is the 2×2 identity matrix. [4]
- Q2.** (i) Solve the inequation $-2 < \frac{3x-1}{2} \leq 4$, where $x \in \mathbb{Z}$ (integers), and write the solution set. [3]
- (ii) Solve the quadratic equation $5x^2 - 3x - 2 = 0$ by the method of factorisation. [4]
- (iii) Find the mean proportional between 9 and 16. [3]
- Q3.** (i) Find the remainder when $x^3 - 2x^2 + 3x - 5$ is divided by $(x - 2)$. [3]
- (ii) The sum of the first 6 terms of an A.P. is 42, and the ratio of its 10th term to its 30th term is $1 : 3$. Find the first term, the common difference, and the 13th term. [4]
- (iii) The 2nd term of a G.P. is 6 and its 5th term is 48. Find the first term and the common ratio. [3]
- Q4.** (i) The point $A(-3, 2)$ is reflected in the origin to A' , and A' is then reflected in the x -axis to A'' . Write the coordinates of A' and A'' , and name the single transformation that maps A to A'' . [4]
- (ii) The mid-point of the line segment joining $A(2a, 4)$ and $B(-2, 3b)$ is $(3, 5)$. Find the values of a and b . [3]
- (iii) Prove that $\frac{1 + \tan^2 \theta}{1 + \cot^2 \theta} = \tan^2 \theta$. [3]

SECTION B**40 Marks — Attempt any FOUR questions***Attempt any four questions from this Section.*

- Q5.** (i) Find the equation of the line with slope -2 and y -intercept 5 . [3]
(ii) Find the equation of the line passing through the points $(3, 4)$ and $(-1, 2)$. [3]
(iii) Find the value of p if the line $2x + py = 5$ is (a) parallel to, and (b) perpendicular to, the line $4x - 2y = 3$. [4]
- Q6.** (i) In $\triangle PQR$, $ST \parallel QR$ with S on PQ and T on PR . If $PS = 3$ cm, $SQ = 4.5$ cm and $PT = 5$ cm, find TR . [4]
(ii) The areas of two similar triangles are 48 cm^2 and 108 cm^2 . If an altitude of the smaller triangle is 4 cm, find the corresponding altitude of the larger triangle. [3]
(iii) In $\triangle ABC$, D and E lie on AB and AC with $DE \parallel BC$ and $AD : DB = 3 : 2$. If the area of $\triangle ADE$ is 18 cm^2 , find the area of $\triangle ABC$. [3]
- Q7.** (i) PQ is a diameter of a circle and R is a point on the circle. If $\angle QPR = 35^\circ$, find $\angle PRQ$ and $\angle PQR$. [4]
(ii) PT is a tangent from an external point P to a circle with centre O and radius 5 cm. If $OP = 13$ cm, find PT . [3]
(iii) PT is a tangent and PAB a secant to a circle drawn from an external point P . If $PT = 6$ cm and $PA = 4$ cm, find AB . [3]
- Q8.** (i) A bag contains 5 red, 3 white and 2 black balls. One ball is drawn at random. Find the probability that it is (a) red, (b) not black. [3]
(ii) Find the maturity value of a recurring deposit of Rs. 500 per month for 2 years at 6% per annum. [3]
(iii) The product of two consecutive positive even numbers is 224. Form a quadratic equation and find the numbers. [4]
- Q9.** (i) Which term of the A.P. $5, 9, 13, \dots$ is 81? [3]
(ii) Find the sum of the first 6 terms of the G.P. $2, 6, 18, \dots$ [3]
(iii) The sum of three numbers in A.P. is 15 and their product is 105. Find the numbers. [4]
- Q10.** (i) Find the values of x and y if $\begin{bmatrix} 2 & x \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ y \end{bmatrix}$. [4]
(ii) If $x : y = 2 : 3$, find the value of $(5x - 2y) : (x + y)$. [3]
(iii) A trader in U.P. buys goods worth Rs. 10000 within U.P. and sells them within U.P. for Rs. 13000, GST being charged at 5% on each transaction. Find the net GST (SGST) paid by the trader to the State Government. [3]
- Q11.** (i) Prove that $(\sec \theta - \cos \theta)(\operatorname{cosec} \theta - \sin \theta) = \frac{1}{\tan \theta + \cot \theta}$. [4]
(ii) The point $P(5, -2)$ is reflected in the x -axis to P' , and P' is reflected in the y -axis to P'' . Write the coordinates of P' and P'' . [3]
(iii) An arc of a circle subtends an angle of 140° at the centre. Find the angle subtended by this arc in the major segment. [3]

ANSWER KEY & MARKING SCHEME

SECTION A

Answers

- Q1.** (i) Value addition = 2000; GST paid to government = 12% of 2000 = Rs. 240. (ii) $n = 18$.
 $I = 800 \cdot \frac{18 \times 19}{2 \times 12} \cdot \frac{10}{100} = 800 \times 14.25 \times 0.1 = \text{Rs. } 1140$. Maturity = $800 \times 18 + 1140 = \text{Rs. } 15540$. (iii)
 $A^2 = \begin{bmatrix} 1 & -8 \\ 16 & -7 \end{bmatrix}$, $2A = \begin{bmatrix} 6 & -4 \\ 8 & 2 \end{bmatrix}$; $A^2 - 2A + I = \begin{bmatrix} -4 & -4 \\ 8 & -8 \end{bmatrix}$.
- Q2.** (i) $-2 < \frac{3x-1}{2} \Rightarrow x > -1$; $\frac{3x-1}{2} \leq 4 \Rightarrow x \leq 3$. So $-1 < x \leq 3$; with $x \in \mathbb{Z}$: $\{0, 1, 2, 3\}$. (ii)
 $5x^2 - 3x - 2 = (5x+2)(x-1) = 0 \Rightarrow x = 1$ or $x = -\frac{2}{5}$. (iii) Mean proportional = $\sqrt{9 \times 16} = 12$.
- Q3.** (i) $f(2) = 8 - 8 + 6 - 5 = 1$; remainder = 1. (ii) $2a + 5d = 14$ (from $S_6 = 42$) and $\frac{a+9d}{a+29d} = \frac{1}{3} \Rightarrow a = d$;
 so $7a = 14$, $a = d = 2$. $a_{13} = a + 12d = 26$. (iii) $ar = 6$, $ar^4 = 48 \Rightarrow r^3 = 8$, $r = 2$, $a = 3$.
- Q4.** (i) $A' = (3, -2)$; $A'' = (3, 2)$; single transformation is reflection in the y -axis. (ii) $\frac{2a-2}{2} = 3 \Rightarrow a = 4$;
 $\frac{4+3b}{2} = 5 \Rightarrow b = 2$. (iii) $\frac{1+\tan^2 \theta}{1+\cot^2 \theta} = \frac{\sec^2 \theta}{\operatorname{cosec}^2 \theta} = \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta$.

SECTION B

Answers

- Q5.** (i) $y = -2x + 5$, i.e. $2x + y - 5 = 0$. (ii) Slope = $\frac{2-4}{-1-3} = \frac{1}{2}$; $y - 4 = \frac{1}{2}(x - 3) \Rightarrow x - 2y + 5 = 0$. (iii)
 Slope of $2x + py = 5$ is $-\frac{2}{p}$; slope of $4x - 2y = 3$ is 2. (a) Parallel: $-\frac{2}{p} = 2 \Rightarrow p = -1$. (b) Perpendicular:
 $-\frac{2}{p} \times 2 = -1 \Rightarrow p = 4$.
- Q6.** (i) $\frac{PS}{SQ} = \frac{PT}{TR} \Rightarrow \frac{3}{4.5} = \frac{5}{TR} \Rightarrow TR = 7.5$ cm. (ii) Ratio of sides = $\sqrt{48:108} = \sqrt{4:9} = 2:3$; altitude
 = $4 \times \frac{3}{2} = 6$ cm. (iii) $\frac{AD}{AB} = \frac{3}{5}$, ratio of areas = $\frac{9}{25}$; area $\triangle ABC = 18 \times \frac{25}{9} = 50 \text{ cm}^2$.
- Q7.** (i) Angle in a semicircle $\angle PRQ = 90^\circ$; $\angle PQR = 180^\circ - 90^\circ - 35^\circ = 55^\circ$. (ii)
 $PT = \sqrt{OP^2 - r^2} = \sqrt{169 - 25} = 12$ cm. (iii) $PT^2 = PA \cdot PB \Rightarrow 36 = 4 \times PB \Rightarrow PB = 9$;
 $AB = PB - PA = 5$ cm.
- Q8.** (i) $P(\text{red}) = \frac{5}{10} = \frac{1}{2}$; $P(\text{not black}) = \frac{8}{10} = \frac{4}{5}$. (ii) $n = 24$;
 $I = 500 \cdot \frac{24 \times 25}{24} \cdot \frac{6}{100} = 500 \times 25 \times 0.06 = 750$; maturity = $500 \times 24 + 750 = \text{Rs. } 12750$. (iii)
 $n(n+2) = 224 \Rightarrow n^2 + 2n - 224 = 0 \Rightarrow (n-14)(n+16) = 0 \Rightarrow n = 14$; numbers are 14 and 16.
- Q9.** (i) $5 + (n-1)4 = 81 \Rightarrow n-1 = 19 \Rightarrow n = 20$; the 20th term. (ii) $S_6 = \frac{2(3^6 - 1)}{3 - 1} = \frac{2 \times 728}{2} = 728$. (iii)
 Let terms be $a-d, a, a+d$: $3a = 15 \Rightarrow a = 5$; $(5-d)(5)(5+d) = 105 \Rightarrow 25 - d^2 = 21 \Rightarrow d = \pm 2$;
 numbers are 3, 5, 7.
- Q10.** (i) $\begin{bmatrix} 2 & x \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2+2x \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ y \end{bmatrix} \Rightarrow 2+2x = 8 \Rightarrow x = 3, y = 5$. (ii) With $x = 2, y = 3$:
 $(5x-2y):(x+y) = (10-6):(2+3) = 4:5$. (iii) Value addition = $13000 - 10000 = 3000$; total GST to
 government = 5% of 3000 = 150; SGST (half) = Rs. 75.

- Q11.** (i) $\sec \theta - \cos \theta = \frac{\sin^2 \theta}{\cos \theta}$ and $\operatorname{cosec} \theta - \sin \theta = \frac{\cos^2 \theta}{\sin \theta}$; their product = $\sin \theta \cos \theta$. Also
 $\tan \theta + \cot \theta = \frac{1}{\sin \theta \cos \theta}$, so $\frac{1}{\tan \theta + \cot \theta} = \sin \theta \cos \theta$. Hence equal. (ii) $P' = (5, 2)$; $P'' = (-5, 2)$. (iii)
 Angle in the major segment = $\frac{1}{2}(140^\circ) = 70^\circ$.