

**MID-TERM EXAMINATION — Class 10 Mathematics (Set 3)**

Chapters: Reflection, Mid-point & Section Formula, Equation of a Line, Circles, Trigonometric Identities, Heights & Distances, Probability, Measures of Central Tendency, Cylinder Cone & Sphere

Max Marks: 80

Name: _____

Time: 2½ Hours

General Instructions: Answers to this paper must be written on the paper provided separately. You will **not** be allowed to write during the first 15 minutes (reading time). Attempt **all** questions from Section A and **any four** questions from Section B. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks. Mathematical tables and squared paper are provided.

SECTION A**40 Marks — Attempt ALL questions***Attempt all questions from this Section.*

- Q1.** (i) The point $A(5, -3)$ is reflected in the y -axis to A' , and A' is reflected in the origin to A'' . Write the coordinates of A' and A'' , and name the single transformation that maps A to A'' . [4]
- (ii) Find the coordinates of the point that divides the line segment joining $A(-1, 7)$ and $B(4, -3)$ internally in the ratio $2 : 3$. [3]
- (iii) Find the equation of the line passing through the points $(3, -2)$ and $(-1, 6)$. [3]
- Q2.** (i) In a circle with centre O , the inscribed angle $\angle BAC = 35^\circ$ stands on the arc BC . Find the central angle $\angle BOC$ subtended by the same arc. [4]
- (ii) Prove that $\frac{\cos \theta}{1 - \tan \theta} + \frac{\sin \theta}{1 - \cot \theta} = \sin \theta + \cos \theta$. [3]
- (iii) The angle of elevation of the top of a tree from a point 20 m from its base on level ground is 30° . Find the height of the tree (in surd form). [3]
- Q3.** (i) A bag contains 3 red, 5 blue and 2 white balls. One ball is drawn at random. Find the probability that it is (a) not white, (b) blue. [3]
- (ii) Find the mean and the median of the data: 14, 16, 18, 20, 22, 24. [4]
- (iii) Find the volume of a cone of base radius 3 cm and height 7 cm. (Take $\pi = \frac{22}{7}$.) [3]
- Q4.** (i) Find the total surface area of a solid cylinder of radius 7 cm and height 10 cm. (Take $\pi = \frac{22}{7}$.) [4]
- (ii) Find the coordinates of the mid-point of the line segment joining $A(-5, 3)$ and $B(3, -7)$. [3]
- (iii) Prove that $\sec^2 A + \operatorname{cosec}^2 A = \sec^2 A \cdot \operatorname{cosec}^2 A$. [3]

SECTION B**40 Marks — Attempt any FOUR questions***Attempt any four questions from this Section.*

- Q5.** (i) Find the equation of the line passing through the points $(4, -1)$ and $(-2, 5)$. [3]
(ii) Find the equation of the line parallel to $3x - y = 7$ and passing through the point $(0, -2)$. [3]
(iii) The line $2x + 3y = 6$ meets the x -axis at A and the y -axis at B . Find the coordinates of A and B , the length AB , and the mid-point of AB . [4]
- Q6.** (i) From an external point P , two tangents PA and PB are drawn to a circle with centre O and radius 8 cm. If $OP = 17$ cm, find the length of each tangent and the perimeter of quadrilateral $OAPB$. [4]
(ii) In a cyclic quadrilateral $PQRS$, $\angle P = (3x)^\circ$ and $\angle R = (2x + 20)^\circ$. Find the value of x and hence $\angle P$. [3]
(iii) Two chords AB and CD of a circle intersect internally at P . If $PA = 10$ cm, $PB = 3$ cm and $PC = 5$ cm, find PD . [3]
- Q7.** (i) A tree is broken by the wind so that its top touches the ground and makes an angle of 30° with the ground at a point 8 m from the foot of the tree. Find the original height of the tree (in surd form). [5]
(ii) The angle of elevation of the top of a tower from a point A on the ground is 30° . On walking 40 m towards the tower to a point B , the angle of elevation becomes 60° . Find the height of the tower (in surd form). [5]
- Q8.** (i) Find the mean of the following frequency distribution: values 10, 20, 30, 40, 50 with frequencies 5, 8, 14, 8, 5 respectively. [5]
(ii) Find the median of the data: 3, 5, 7, 9, 11, 13. [3]
(iii) Find the mode of the data: 4, 4, 5, 6, 4, 7, 6. [2]
- Q9.** (i) A solid metallic sphere of radius 3 cm is melted and recast into a solid cone of base radius 6 cm. Find the height of the cone. [4]
(ii) Find the total surface area of a solid hemisphere of radius 7 cm. (Take $\pi = \frac{22}{7}$.) [3]
(iii) Find the volume of a solid cylinder of radius 7 cm and height 15 cm. (Take $\pi = \frac{22}{7}$.) [3]
- Q10.** (i) The point $P(4, 2)$ is reflected in the x -axis to P' , and P is reflected in the origin to P'' . Write the coordinates of P' and P'' , and find the area of $\triangle PP'P''$. [4]
(ii) Find the ratio in which the y -axis divides the line segment joining $A(-3, 4)$ and $B(6, -2)$. [3]
(iii) Find the equation of the perpendicular bisector of the line segment joining $A(2, 3)$ and $B(6, 7)$. [3]
- Q11.** (i) Prove that $\frac{\sin A}{\cot A + \operatorname{cosec} A} = 2 + \frac{\sin A}{\cot A - \operatorname{cosec} A}$. [4]
(ii) Prove that $\tan^2 A - \sin^2 A = \tan^2 A \cdot \sin^2 A$. [3]
(iii) Two dice are thrown together once. Find the probability of getting (a) a product of 12, (b) a sum of 8. [3]

ANSWER KEY & MARKING SCHEME

SECTION A

Answers

- Q1.** (i) $A' = (-5, -3)$; $A'' = (5, 3)$; single transformation is reflection in the x -axis. (ii) Point $= \left(\frac{2(4) + 3(-1)}{5}, \frac{2(-3) + 3(7)}{5} \right) = \left(\frac{5}{5}, \frac{15}{5} \right) = (1, 3)$. (iii) Slope $= \frac{6 - (-2)}{-1 - 3} = -2$; $y + 2 = -2(x - 3) \Rightarrow 2x + y - 4 = 0$.
- Q2.** (i) Central angle $= 2 \times$ inscribed angle $= 2(35^\circ) = 70^\circ$. (ii) $\frac{\cos \theta}{1 - \tan \theta} + \frac{\sin \theta}{1 - \cot \theta} = \frac{\cos^2 \theta}{\cos \theta - \sin \theta} + \frac{\sin^2 \theta}{\sin \theta - \cos \theta} = \frac{\cos^2 \theta - \sin^2 \theta}{\cos \theta - \sin \theta} = \cos \theta + \sin \theta$. (iii) $h = 20 \tan 30^\circ = \frac{20}{\sqrt{3}} \text{ m} \approx 11.55 \text{ m}$.
- Q3.** (i) $P(\text{not white}) = \frac{8}{10} = \frac{4}{5}$; $P(\text{blue}) = \frac{5}{10} = \frac{1}{2}$. (ii) Mean $= \frac{14 + 16 + 18 + 20 + 22 + 24}{6} = \frac{114}{6} = 19$; median $= \frac{18 + 20}{2} = 19$. (iii) $V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 9 \times 7 = 66 \text{ cm}^3$.
- Q4.** (i) TSA $= 2\pi r(h + r) = 2 \times \frac{22}{7} \times 7 \times (10 + 7) = 2 \times 22 \times 17 = 748 \text{ cm}^2$. (ii) Mid-point $= \left(\frac{-5 + 3}{2}, \frac{3 - 7}{2} \right) = (-1, -2)$. (iii) $\sec^2 A + \operatorname{cosec}^2 A = \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} = \frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A} = \frac{1}{\sin^2 A \cos^2 A} = \sec^2 A \cdot \operatorname{cosec}^2 A$.

SECTION B

Answers

- Q5.** (i) Slope $= \frac{5 - (-1)}{-2 - 4} = -1$; $y + 1 = -(x - 4) \Rightarrow x + y - 3 = 0$. (ii) Slope of $3x - y = 7$ is 3; $y + 2 = 3(x - 0) \Rightarrow 3x - y - 2 = 0$. (iii) $A = (3, 0)$, $B = (0, 2)$; $AB = \sqrt{3^2 + 2^2} = \sqrt{13}$ units; mid-point $= \left(\frac{3}{2}, 1 \right)$.
- Q6.** (i) $PA = PB = \sqrt{OP^2 - r^2} = \sqrt{289 - 64} = 15$ cm; perimeter of $OAPB = 2(8 + 15) = 46$ cm. (ii) Opposite angles: $3x + (2x + 20) = 180 \Rightarrow 5x = 160 \Rightarrow x = 32$; $\angle P = 3(32) = 96^\circ$. (iii) $10 \times 3 = 5 \times PD \Rightarrow PD = 6$ cm.
- Q7.** (i) Let the broken part make 30° at 8 m from the foot. Height of break $= 8 \tan 30^\circ = \frac{8}{\sqrt{3}}$; length of broken part $= \frac{8}{\cos 30^\circ} = \frac{16}{\sqrt{3}}$. Original height $= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}} = 8\sqrt{3} \text{ m} \approx 13.86 \text{ m}$. (ii) Let $BC = x$ (from foot to B), height $= h$. $\tan 60^\circ = \frac{h}{x} \Rightarrow h = \sqrt{3}x$; $\tan 30^\circ = \frac{h}{x + 40} \Rightarrow h = \frac{x + 40}{\sqrt{3}}$. So $\sqrt{3}x = \frac{x + 40}{\sqrt{3}} \Rightarrow 3x = x + 40 \Rightarrow x = 20$; $h = 20\sqrt{3} \text{ m} \approx 34.64 \text{ m}$.
- Q8.** (i) $\sum fx = 50 + 160 + 420 + 320 + 250 = 1200$, $\sum f = 40$; mean $= \frac{1200}{40} = 30$. (ii) $n = 6$; median $= \frac{7 + 9}{2} = 8$. (iii) Mode $= 4$.
- Q9.** (i) Volume of sphere $= \frac{4}{3} \pi (3)^3 = 36\pi$; cone volume $= \frac{1}{3} \pi (6)^2 h = 12\pi h$; $12\pi h = 36\pi \Rightarrow h = 3$ cm. (ii) TSA of hemisphere $= 3\pi r^2 = 3 \times \frac{22}{7} \times 49 = 462 \text{ cm}^2$. (iii) $V = \pi r^2 h = \frac{22}{7} \times 49 \times 15 = 2310 \text{ cm}^3$.

- Q10.** (i) $P' = (4, -2)$; $P'' = (-4, -2)$. In $\triangle PP'P''$: $PP' = 4$ (vertical) and horizontal distance from P'' to line $x = 4$ is 8; area = $\frac{1}{2} \times 4 \times 8 = 16$ sq units. (ii) On the y -axis $x = 0$; ratio $k : 1$: $\frac{6k-3}{k+1} = 0 \Rightarrow k = \frac{1}{2}$; ratio = $1 : 2$. (iii) Mid-point = $(4, 5)$; slope of $AB = 1$, perpendicular slope = -1 ; $y - 5 = -(x - 4) \Rightarrow x + y - 9 = 0$.
- Q11.** (i) Bringing the RHS to one side and simplifying using $\cot^2 A - \operatorname{cosec}^2 A = -1$ verifies the identity; both sides reduce to $\frac{\sin A}{\cot A + \operatorname{cosec} A}$. (ii)
- $$\tan^2 A - \sin^2 A = \frac{\sin^2 A}{\cos^2 A} - \sin^2 A = \frac{\sin^2 A(1 - \cos^2 A)}{\cos^2 A} = \frac{\sin^2 A \cdot \sin^2 A}{\cos^2 A} = \tan^2 A \sin^2 A. \text{ (iii) Product}$$
- 12: $(2, 6), (6, 2), (3, 4), (4, 3), P = \frac{4}{36} = \frac{1}{9}$; sum 8: $(2, 6), (3, 5), (4, 4), (5, 3), (6, 2), P = \frac{5}{36}$.