KUMAR CLASSES

NCERT SOLUTIONS · CLASS 12 MATHEMATICS

Miscellaneous Exercise

Chapter 7 — Integrals

Q1–23: integrate the functions. Q24–31: evaluate the definite integrals. Q32–37: prove the given results. Q38–40: choose the correct option.

1. $\int \frac{1}{x-x^3}\,dx$

Write $\frac{1}{x(1-x^2)} = \frac{1}{x} + \frac{x}{1-x^2}$.

Sol. $= \frac{1}{2} \log\left| \frac{x^2}{1-x^2} \right| + C$

2. $\int \frac{1}{\sqrt{x+a} + \sqrt{x+b}}\,dx$

Rationalise by multiplying with $\sqrt{x+a} - \sqrt{x+b}$ (denominator becomes $a-b$).

Sol. $= \frac{2}{3(a-b)} \left[(x+a)^{3/2} - (x+b)^{3/2} \right] + C$

3. $\int \frac{1}{x\sqrt{ax-x^2}}\,dx$

Put $x = \frac{a}{t}$ (given); reduces to a $\sqrt{a^2-u^2}$-type integral.

Sol. $= -\frac{2}{a}\sqrt{\frac{a-x}{x}} + C$

4. $\int \frac{1}{x^2(x^4+1)^{3/4}}\,dx$

Take x^4 out of the root; put $t = 1 + x^{-4}$.

Sol. $= -\left(\frac{x^4+1}{x^4} \right)^{1/4} + C$

5. $\int \frac{1}{x^{1/2} + x^{1/3}}\,dx$

Put $x = t^6$ (given); integral becomes $\int \frac{6t^3}{t+1}\,dt$.

Sol. $= 2\sqrt{x} - 3x^{1/3} + 6x^{1/6} - 6\log(x^{1/6}+1) + C$

6. $\int \dfrac{5x}{(x+1)(x^2+9)}\,dx$

Partial fractions: $\dfrac{-1/2}{x+1} + \dfrac{(1/2)x + 9/2}{x^2+9}$.

Sol. $= -\dfrac{1}{2}\log|x+1| + \dfrac{1}{4}\log(x^2+9) + \dfrac{3}{2}\tan^{-1}\dfrac{x}{3} + C$

7. $\int \dfrac{\sin x}{\sin(x-a)}\,dx$

Put $x-a = t$; then $\sin x = \sin(t+a) = \sin t \cos a + \cos t \sin a$.

Sol. $= x\cos a + \sin a \cdot \log|\sin(x-a)| + C$

8. $\int \dfrac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}\,dx$

Use $e^{k\log x} = x^k$: $= \dfrac{x^5-x^4}{x^3-x^2} = x^2$.

Sol. $= \dfrac{x^3}{3} + C$

9. $\int \dfrac{\cos x}{\sqrt{4-\sin^2 x}}\,dx$

Put $t = \sin x$ $(dt = \cos x\,dx)$; $\sqrt{a^2-t^2}$ form.

Sol. $= \sin^{-1}\left(\dfrac{\sin x}{2}\right) + C$

10. $\int \dfrac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x\cos^2 x}\,dx$

Factor: $\sin^8 x - \cos^8 x = -\cos 2x \cdot (1-2\sin^2 x\cos^2 x)$.

Sol. $= \int(-\cos 2x)\,dx = -\dfrac{1}{2}\sin 2x + C$

11. $\int \dfrac{1}{\cos(x+a)\cos(x+b)}\,dx$

Use $\sin[(x+a)-(x+b)] = \sin(a-b)$ to split into tangents.

Sol. $= \dfrac{1}{\sin(a-b)}\log\left|\dfrac{\cos(x+b)}{\cos(x+a)}\right| + C$

12. $\int \dfrac{x^3}{\sqrt{1-x^8}}\,dx$

Put $t = x^4$ $(dt = 4x^3 dx)$; $\sqrt{1-t^2}$ form.

Sol. $= \dfrac{1}{4}\sin^{-1}(x^4) + C$

13. $\int \frac{e^x}{(1+e^x)(2+e^x)}\,dx$

Put $t = e^x$; partial fractions $\frac{1}{1+t} - \frac{1}{2+t}$.

Sol. $= \log\left|\frac{1+e^x}{2+e^x}\right| + C$

14. $\int \frac{1}{(x^2+1)(x^2+4)}\,dx$

Partial fractions: $\frac{1}{3}\left[\frac{1}{x^2+1} - \frac{1}{x^2+4}\right]$.

Sol. $= \frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\frac{x}{2} + C$

15. $\int \cos^3 x \cdot e^{\log \sin x}\,dx$

$e^{\log \sin x} = \sin x$; put $t = \cos x$.

Sol. $= \int \cos^3 x \sin x\,dx = -\frac{1}{4}\cos^4 x + C$

16. $\int e^{3\log x}(x^4+1)^{-1}\,dx$

$e^{3\log x} = x^3$, so integrand $= \frac{x^3}{x^4+1}$.

Sol. $= \frac{1}{4}\log(x^4+1) + C$

17. $\int f'(ax+b)\,[f(ax+b)]^n\,dx$

Put $u = f(ax+b)$, so $du = a\cdot f'(ax+b)dx$.

Sol. $= \frac{[f(ax+b)]^{n+1}}{a(n+1)} + C$

18. $\int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}}\,dx$

$\sin(x+\alpha) = \sin x\cos\alpha + \cos x\sin\alpha$; factor $\sin x$ and put $t = \cos\alpha + \sin\alpha\cot x$.

Sol. $= -\frac{2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}} + C$

19. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$

Put $\sqrt{x} = \cos\theta$; the radical simplifies to $\tan\frac{\theta}{2}$.

Sol. $= \cos^{-1}\sqrt{x} - 2\sqrt{1-x} + \sqrt{x(1-x)} + C$

20. $\int \frac{2 + \sin 2x}{1 + \cos 2x} e^x \, dx$

Simplify $\frac{2+\sin 2x}{1+\cos 2x} = \tan x + \sec^2 x$; *then* $e^x[f + f']$ *with* $f = \tan x$.

Sol. $= e^x \tan x + C$

21. $\int \frac{x^2 + x + 1}{(x+1)^2(x+2)} \, dx$

Partial fractions (repeated factor): $\frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{3}{x+2}$.

Sol. $= -2 \log|x+1| - \frac{1}{x+1} + 3 \log|x+2| + C$

22. $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}} \, dx$

Put $x = \cos\theta$; *the argument becomes* $\tan\frac{\theta}{2}$, *so the expression* $= \frac{1}{2}\cos^{-1}x$.

Sol. $= \frac{1}{2}\left(x \cos^{-1}x - \sqrt{1-x^2} \right) + C$

23. $\int \frac{\sqrt{x^2+1}\,[\log(x^2+1) - 2\log x]}{x^4} \, dx$

$= \sqrt{1+\frac{1}{x^2}} \cdot \frac{\log(1+1/x^2)}{x^3}$; *put* $t = 1 + \frac{1}{x^2}$.

Sol. $= -\frac{1}{3}\left(1 + \frac{1}{x^2}\right)^{3/2} \left[\log\left(1+\frac{1}{x^2}\right) - \frac{2}{3}\right] + C$

24. $\int_{\pi/2}^{\pi} e^x\left(\frac{1-\sin x}{1-\cos x}\right) dx$

$e^x[f + f']$ *with* $f = -\cot\frac{x}{2}$.

Sol. $= \left[-e^x \cot\frac{x}{2}\right]_{\pi/2}^{\pi} = e^{\pi/2}$

25. $\int_0^{\pi/4} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} \, dx$

Divide by $\cos^4 x$; *put* $t = \tan^2 x$.

Sol. $= \frac{1}{2}\left[\tan^{-1}(\tan^2 x)\right]_0^{\pi/4} = \frac{\pi}{8}$

26. $\int_0^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x} \, dx$

Reduces (via $t = \tan x$*) to a standard* $\tan^{-1}$ *evaluation.*

Sol. $= \frac{\pi}{6}$

27. $\int_{\pi/6}^{\pi/3} \dfrac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx$

$\sin 2x = 1 - (\sin x - \cos x)^2$; put $t = \sin x - \cos x$.

Sol. $= [\, \sin^{-1}(\sin x - \cos x)\,]_{\pi/6}^{\pi/3} = 2 \sin^{-1} \dfrac{\sqrt{3}-1}{2}$

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28. $\int_0^1 \dfrac{1}{\sqrt{1+x} - \sqrt{x}}$

Rationalise the denominator (it becomes 1): $= \int_0^1 (\sqrt{1+x} + \sqrt{x})\, dx$.

Sol. $= \dfrac{4\sqrt{2}}{3}$

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29. $\int_0^{\pi/4} \dfrac{\sin x + \cos x}{9 + 16 \sin 2x}\, dx$

Put $t = \sin x - \cos x$; then $16 \sin 2x = 16 - 16t^2$, so denom $= 25 - 16t^2$.

Sol. $= [\, \dfrac{1}{40} \log\left| \dfrac{5+4t}{5-4t} \right| \,] = \dfrac{1}{20} \log 3$

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30. $\int_0^{\pi/2} \sin 2x \cdot \tan^{-1}(\sin x)\, dx$

Put $u = \sin x$; $= 2\int_0^1 u \tan^{-1} u\, du$ (by parts).

Sol. $= \dfrac{\pi}{2} - 1$

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31. $\int_1^4 [\, |x-1| + |x-2| + |x-3|\,]\, dx$

Integrate each modulus over its sign-pieces and add.

Sol. $= \dfrac{9}{2} + \dfrac{5}{2} + \dfrac{5}{2} = \dfrac{19}{2}$

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32. **Prove:** $\int_1^3 \dfrac{dx}{x^2(x+1)} = \dfrac{2}{3} + \log \dfrac{2}{3}$

Partial fractions: $\dfrac{1}{x^2(x+1)} = -\dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x+1}$.

Sol. $= [\, \log \dfrac{x+1}{x} - \dfrac{1}{x}\,]_1^3 = \log \dfrac{2}{3} + \dfrac{2}{3}$ (proved)

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33. **Prove:** $\int_0^1 x\, e^x\, dx = 1$

By parts: $\int x\, e^x dx = (x-1)e^x$.

Sol. $= [\, (x-1)e^x\,]_0^1 = 0 - (-1) = 1$ (proved)

34. Prove: $\int_{-1}^{1} x^{17}\cos^4 x\,dx = 0$

$x^{17}\cos^4 x$ is odd (odd × even) over a symmetric interval.

Sol. $= 0$ (proved)

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35. Prove: $\int_0^{\pi/2} \sin^3 x\,dx = \frac{2}{3}$

$\sin^3 x = \sin x(1-\cos^2 x)$; antiderivative $-\cos x + \frac{\cos^3 x}{3}$.

Sol. $= \left[-\cos x + \frac{\cos^3 x}{3} \right]_0^{\pi/2} = 0 - \left(-\frac{2}{3}\right) = \frac{2}{3}$ (proved)

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36. Prove: $\int_0^{\pi/4} 2\tan^3 x\,dx = 1 - \log 2$

$\tan^3 x = \tan x(\sec^2 x - 1)$; antiderivative $\tan^2 x + 2\log|\cos x|$.

Sol. $= \left[\tan^2 x + 2\log|\cos x| \right]_0^{\pi/4} = 1 - \log 2$ (proved)

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37. Prove: $\int_0^1 \sin^{-1} x\,dx = \frac{\pi}{2} - 1$

By parts (1 as second function): $\int \sin^{-1} x\,dx = x\sin^{-1} x + \sqrt{1-x^2}$.

Sol. $= \left[x\sin^{-1} x + \sqrt{1-x^2} \right]_0^1 = \frac{\pi}{2} - 1$ (proved)

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38. $\int \frac{dx}{e^x + e^{-x}}$ **is equal to**

Multiply by e^x: $= \int \frac{e^x dx}{e^{2x}+1}$; put $t = e^x$.

Sol. $= \tan^{-1}(e^x) + C$

∴ Option (A)

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39. $\int \frac{\cos 2x}{(\sin x + \cos x)^2}\,dx$ **is equal to**

$\cos 2x = (\cos x - \sin x)(\cos x + \sin x)$; cancel one factor.

Sol. $= \log|\sin x + \cos x| + C$

∴ Option (B)

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40. If $f(a+b-x) = f(x)$, then $\int_a^b x\, f(x)\, dx$ is equal to

Apply property P_4 to I and add; $2I = (a+b)\int_a^b f(x)dx$.

Sol. $= \dfrac{a+b}{2} \int_a^b f(x)\, dx$

$\therefore$ Option (D)