KUMAR CLASSES

NCERT SOLUTIONS · CLASS 12 MATHEMATICS

Exercise 7.10

Chapter 7 — Integrals · Properties of Definite Integrals

Q1–19: evaluate using the properties P_1–P_7 (especially the reflection property P_4). Q20–21: choose the correct option.

1. $\int_0^{\pi/2} \cos^2 x \, dx$

Use $\cos^2 x = \frac{1+\cos 2x}{2}$.

Sol. $= \frac{1}{2}\left[x + \frac{\sin 2x}{2} \right]_0^{\pi/2} = \frac{\pi}{4}$

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2. $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx$

Property P_4 ($x \to \frac{\pi}{2} - x$) swaps sin ↔ cos; adding gives $2I = \int_0^{\pi/2} dx$.

Sol. $2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

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3. $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} \, dx$

Same P_4 trick as Q2.

Sol. $2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

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4. $\int_0^{\pi/2} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} \, dx$

Same P_4 trick.

Sol. $2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

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5. $\int_{-5}^{5} |x+2| \, dx$

$|x+2| = -(x+2)$ on $[-5,-2]$ and $(x+2)$ on $[-2,5]$; split.

Sol. $= \int_{-5}^{-2} -(x+2)dx + \int_{-2}^{5} (x+2)dx = \frac{9}{2} + \frac{49}{2} = 29$

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6. $\int_2^8 |x-5|\, dx$

Split at $x = 5$.

Sol. $= \int_2^5 (5-x)dx + \int_5^8 (x-5)dx = \frac{9}{2} + \frac{9}{2} = \mathbf{9}$

7. $\int_0^1 x(1-x)^n\, dx$

By P_4 ($x \to 1-x$): $= \int_0^1 (1-x)x^n dx = \int (x^n - x^{n+1})dx$.

Sol. $= \frac{1}{n+1} - \frac{1}{n+2} = \mathbf{\frac{1}{(n+1)(n+2)}}$

8. $\int_0^{\pi/4} \log(1+\tan x)\, dx$

By P_4 ($x \to \frac{\pi}{4} - x$): $\log(1+\tan(\frac{\pi}{4} - x)) = \log \frac{2}{1+\tan x}$; add.

Sol. $2I = \frac{\pi}{4} \log 2 \Rightarrow \mathbf{I = \frac{\pi}{8} \log 2}$

9. $\int_0^2 x\sqrt{2-x}\, dx$

By P_4 ($x \to 2-x$): $= \int_0^2 (2-x)\sqrt{x}\, dx$.

Sol. $= \int_0^2 (2x^{1/2} - x^{3/2})\, dx = \mathbf{\frac{16\sqrt{2}}{15}}$

10. $\int_0^{\pi/2} (2 \log \sin x - \log \sin 2x)\, dx$

$\log \sin 2x = \log 2 + \log \sin x + \log \cos x$, and $\int_0^{\pi/2} \log \sin x\, dx = -\frac{\pi}{2} \log 2$.

Sol. $= \int_0^{\pi/2} (\log \sin x - \log \cos x - \log 2)\, dx = \mathbf{-\frac{\pi}{2} \log 2}$

11. $\int_{-\pi/2}^{\pi/2} \sin^2 x\, dx$

$\sin^2 x$ is even, so $= 2\int_0^{\pi/2} \sin^2 x\, dx$.

Sol. $= 2 \cdot \frac{\pi}{4} = \mathbf{\frac{\pi}{2}}$

12. $\int_0^{\pi} \frac{x}{1+\sin x}\, dx$

By P_3 ($x \to \pi - x$): $2I = \pi \int_0^{\pi} \frac{dx}{1+\sin x}$ and $\int_0^{\pi} \frac{dx}{1+\sin x} = 2$.

Sol. $2I = 2\pi \Rightarrow \mathbf{I = \pi}$

13. $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx$

$\sin^7 x$ is an odd function integrated over a symmetric interval.

Sol. $= \mathbf{0}$

14. $\int_0^{2\pi} \cos^5 x \, dx$

Odd powers of cos over a full period give zero.

Sol. $= \mathbf{0}$

15. $\int_0^{\pi/2} \dfrac{\sin x - \cos x}{1+\sin x \cos x} \, dx$

By P_4 $(x \to \frac{\pi}{2} - x)$ the integrand changes sign, so $I = -I$.

Sol. $2I = 0 \Rightarrow \mathbf{I = 0}$

16. $\int_0^{\pi} \log(1+\cos x) \, dx$

$1+\cos x = 2\cos^2 \frac{x}{2}$, so $\log(1+\cos x) = \log 2 + 2 \log \cos \frac{x}{2}$.

Sol. $= \pi \log 2 - 2\pi \log 2 = \mathbf{-\pi \log 2}$

17. $\int_0^{a} \dfrac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} \, dx$

By P_4 $(x \to a-x)$: $I(x) + I(a-x) = \int_0^a dx = a$.

Sol. $2I = a \Rightarrow \mathbf{I = \dfrac{a}{2}}$

18. $\int_0^4 |x-1| \, dx$

Split at $x = 1$.

Sol. $= \int_0^1 (1-x)dx + \int_1^4 (x-1)dx = \frac{1}{2} + \frac{9}{2} = \mathbf{5}$

19. **Show that $\int_0^a f(x)g(x)dx = 2\int_0^a f(x)dx$, given $f(x)=f(a-x)$ and $g(x)+g(a-x)=4$.**

Apply P_4 to $I = \int_0^a f(x)g(x)dx$ and use $f(a-x)=f(x)$.

Sol. $I = \int_0^a f(a-x)g(a-x)dx = \int_0^a f(x)g(a-x)dx$

$\therefore 2I = \int_0^a f(x)[g(x)+g(a-x)]dx = \int_0^a f(x)\cdot 4\,dx$

$\Rightarrow \mathbf{I = 2\int_0^a f(x)dx}$ (proved)

20. **The value of $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)\,dx$ is**

x^3, $x\cos x$ and $\tan^5 x$ are all odd → their integrals vanish; only $\int 1\,dx$ survives.

Sol. $= \int_{-\pi/2}^{\pi/2} 1\,dx = \boldsymbol{\pi}$

$\therefore$ Option (C)

21. **The value of $\int_0^{\pi/2} \log\left(\dfrac{4+3\sin x}{4+3\cos x}\right) dx$ is**

By P_4 ($x \to \frac{\pi}{2} - x$) sin ↔ cos, so the log inverts and $I = -I$.

Sol. $2I = 0 \Rightarrow \mathbf{I = 0}$

$\therefore$ Option (C)