KUMAR CLASSES

Excellence in Mathematics

CLASS 10 MATHEMATICS

Chapter 10 : CIRCLES – NCERT SOLUTIONS

Complete Concept Notes

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Exercise 10.1

Q1. How many tangents can a circle have?

Solution :

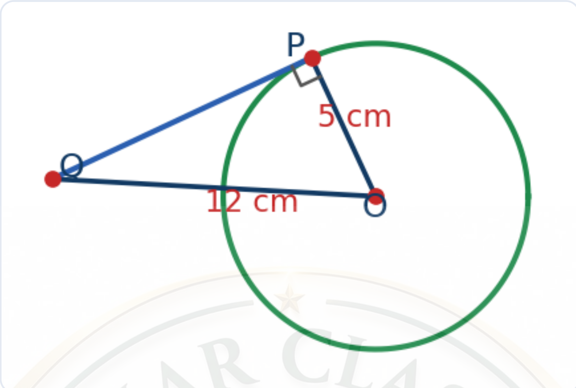
- A circle can have **infinitely many** tangents.
- This is because a circle has infinitely many points on it, and at each point exactly one tangent can be drawn.

Q2. Fill in the blanks:

Solution :

- (i) A tangent to a circle intersects it in **one** point.
- (ii) A line intersecting a circle in two points is called a **secant**.
- (iii) A circle can have **two** parallel tangents at the most.
- (iv) The common point of a tangent to a circle and the circle is called the **point of contact**.

Q3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ=12 cm. Length PQ is:

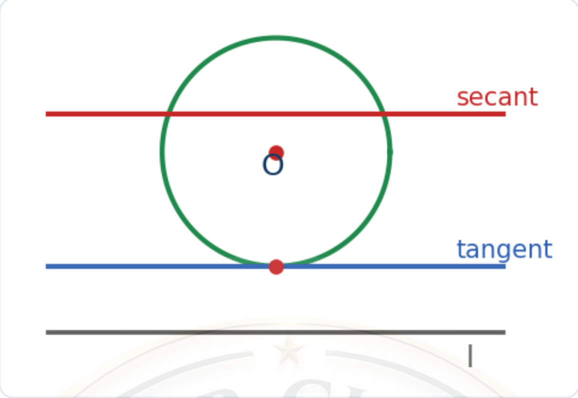


Solution :

- Since PQ is a tangent at P, by Theorem 10.1 $OP \perp PQ$.
- So $\triangle$ OPQ is right-angled at P.
- By Pythagoras Theorem: $OQ^2=OP^2+PQ^2$
- $PQ^2=OQ^2-OP^2=12^2-5^2=144-25=119$
- $PQ=\sqrt{119}$ cm.

Answer : (D) $\sqrt{119}$ cm

Q4. Draw a circle and two lines parallel to a given line such that one is a tangent and the other a secant to the circle.

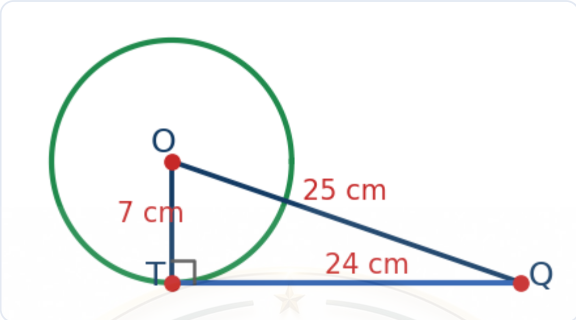


Solution :

- *Draw a circle and a given line l.*
- *Draw a line parallel to l that* ***touches*** *the circle at exactly one point — this is the* ***tangent****.*
- *Draw another line parallel to l that* ***cuts*** *the circle at two points — this is the* ***secant****.*

Exercise 10.2

Q1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:

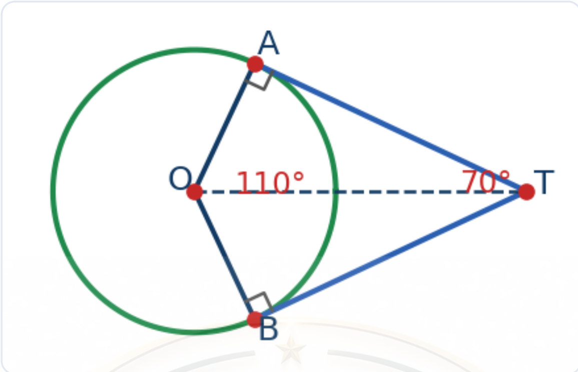


Solution :

- Let the tangent touch the circle at T. Then $OT \perp TQ$ (Theorem 10.1).
- So $\triangle OTQ$ is right-angled at T.
- $OQ^2 = OT^2 + TQ^2 \Rightarrow 25^2 = r^2 + 24^2$
- $r^2 = 625 - 576 = 49 \Rightarrow r = 7$ cm.

Answer : (A) 7 cm

Q2. In the figure, if TP and TQ are two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then $\angle PTQ$ is equal to:

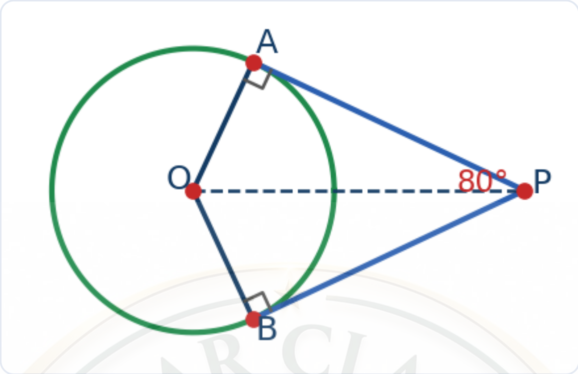


Solution :

- $OP \perp TP$ and $OQ \perp TQ$ (tangent $\perp$ radius).
- In quadrilateral OPTQ: $\angle OPT = \angle OQT = 90^\circ$.
- Sum of angles of a quadrilateral $= 360^\circ$:
- $\angle PTQ = 360^\circ - 90^\circ - 90^\circ - 110^\circ = 70^\circ$.

Answer : (B) 70°

Q3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, then $\angle POA$ is equal to:

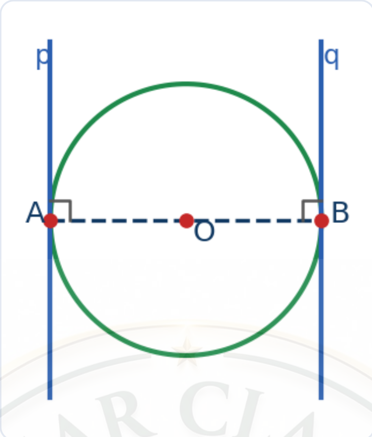


Solution :

- $\angle APB = 80^\circ$; $OA \perp PA$ and $OB \perp PB$.
- In quadrilateral OAPB: $\angle AOB = 360^\circ - 90^\circ - 90^\circ - 80^\circ = 100^\circ$.
- OP bisects $\angle AOB$ (since $\triangle OAP \cong \triangle OBP$).
- $\angle POA = 1/2 \angle AOB = 1/2(100^\circ) = 50^\circ$.

Answer : (A) 50°

Q4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.



Solution :

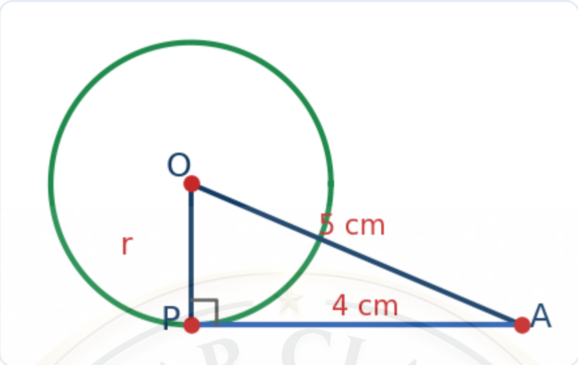
- Let AB be a diameter and let p,q be the tangents at A and B.
- By Theorem 10.1: tangent $p \perp OA$ and tangent $q \perp OB$.
- But OA and OB lie along the same straight line AB (diameter).
- So p and q are both perpendicular to the same line AB.
- Two lines perpendicular to the same line are parallel. $\therefore p \parallel q$. **(Proved)**

Q5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Solution :

- Let XY be a tangent to the circle at P, and let ℓ be the perpendicular to XY at P.
- By Theorem 10.1, the radius $OP \perp XY$ at P.
- Through a given point on a line, there is only **one** perpendicular to that line.
- So OP and ℓ are the same line; hence ℓ passes through the centre O. **(Proved)**

Q6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

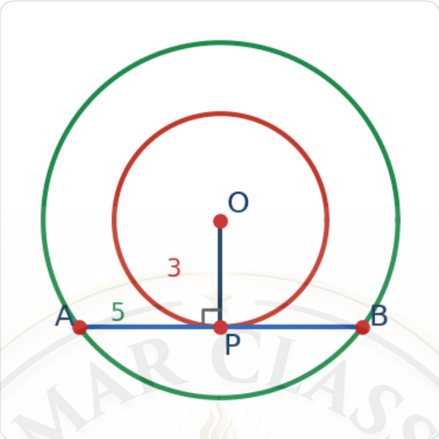


Solution :

- Let the tangent touch at P. Then $OP \perp PA$ (Theorem 10.1).
- In right $\triangle$ OPA: $OA^2=OP^2+PA^2$
- $5^2=r^2+4^2 \Rightarrow r^2=25-16=9$
- $r=3$ cm.

Answer : Radius $=3$ cm

Q7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

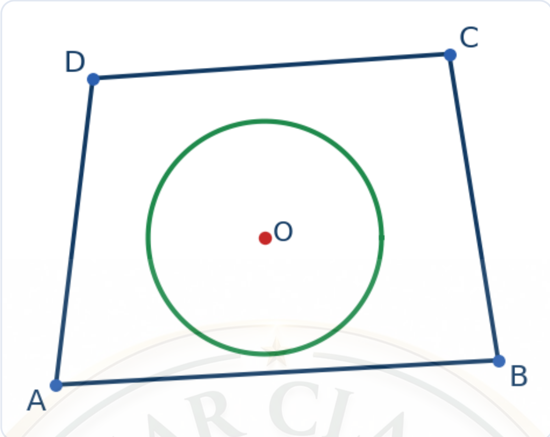


Solution :

- Let O be the common centre and AB the chord of the larger circle touching the smaller circle at P.
- Then $OP \perp AB$ (radius $\perp$ tangent) and $OP=3$ cm.
- The perpendicular from the centre bisects the chord, so $AP=PB$.
- In right $\triangle OPA$: $OA^2=OP^2+AP^2 \Rightarrow 5^2=3^2+AP^2$
- $AP^2=25-9=16 \Rightarrow AP=4$ cm.
- $AB=2\times AP=2\times 4=8$ cm.

Answer : Chord $=8$ cm

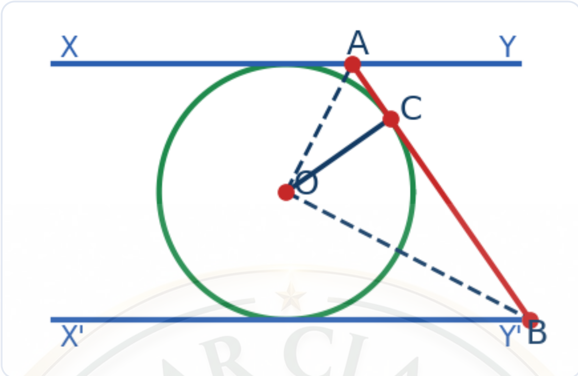
Q8. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB+CD=AD+BC.



Solution :

- Let the circle touch AB,BC,CD,DA at P,Q,R,S respectively.
- Tangents drawn from an external point are equal (Theorem 10.2):
- AP=AS,quad BP=BQ,quad CR=CQ,quad DR=DS.
- Now AB+CD=(AP+BP)+(CR+DR)
- =(AS+BQ)+(CQ+DS)=(AS+DS)+(BQ+CQ)=AD+BC. **(Proved)**

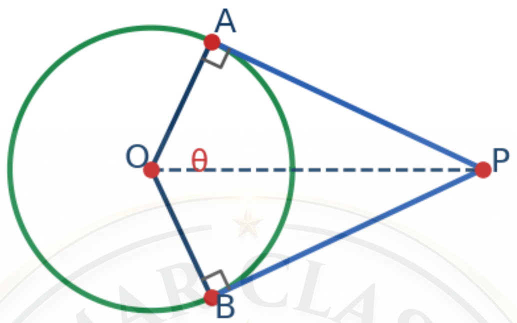
Q9. XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and $X'Y'$ at B. Prove that $\angle AOB = 90^\circ$.



Solution :

- Let XY touch the circle at P and $X'Y'$ touch it at Q; AB touches at C.
- AP and AC are tangents from A, so OA bisects $\angle PAC$; thus $\angle OAC = frac12\angle PAC$.
- Similarly OB bisects $\angle QBC$; thus $\angle OBC = frac12\angle QBC$.
- Since $XY \parallel X'Y'$, the co-interior angles $\angle PAC + \angle QBC = 180^\circ$.
- $\therefore \angle OAC + \angle OBC = frac12(180^\circ) = 90^\circ$.
- In $\triangle OAB$: $\angle AOB = 180^\circ - (\angle OAC + \angle OBC) = 180^\circ - 90^\circ = 90^\circ$. **(Proved)**

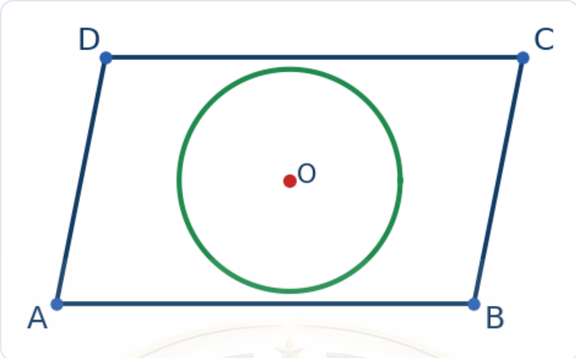
Q10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.



Solution :

- Let PA and PB be the tangents from external point P, touching at A and B.
- $OA \perp PA$ and $OB \perp PB$, so $\angle OAP = \angle OBP = 90^\circ$.
- In quadrilateral OAPB, the angle sum is 360°:
- $\angle APB + \angle AOB + 90^\circ + 90^\circ = 360^\circ$
- $\angle APB + \angle AOB = 180^\circ$ — i.e. they are supplementary. **(Proved)**

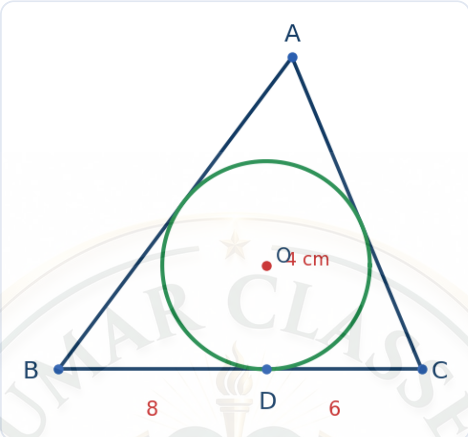
Q11. Prove that the parallelogram circumscribing a circle is a rhombus.



Solution :

- Let parallelogram ABCD circumscribe a circle, touching AB,BC,CD,DA at P,Q,R,S.
- Equal tangents: AP=AS, BP=BQ, CR=CQ, DR=DS.
- As in a circumscribed quadrilateral: AB+CD=AD+BC.
- In a parallelogram AB=CD and AD=BC, so 2AB=2AD⇒ AB=AD.
- A parallelogram with a pair of adjacent sides equal is a **rhombus**. **(Proved)**

Q12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.

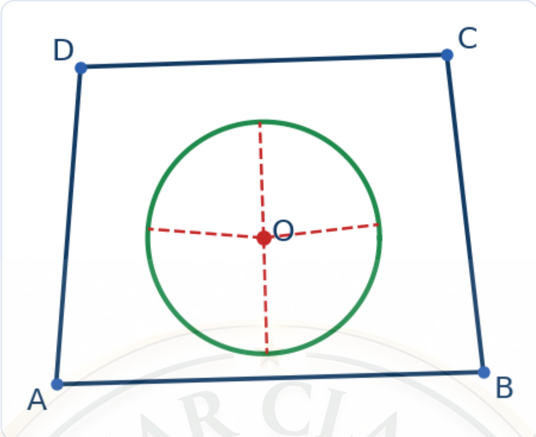


Solution :

- Let the circle touch BC, CA, AB at D, E, F. Using equal tangents:
- $BD=BF=8$, $CD=CE=6$, and let $AF=AE=x$.
- Then $AB=x+8$, $AC=x+6$, $BC=14$; semi-perimeter $s=((x+8)+(x+6)+14)/2=x+14$.
- Area $=r\times s=4(x+14)$.
- Also, by Heron: Area $=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{(x+14)(x)(6)(8)}=\sqrt{48x(x+14)}$.
- Equate: $4(x+14)=\sqrt{48x(x+14)} \Rightarrow 16(x+14)^2=48x(x+14)$
- $16(x+14)=48x \Rightarrow x+14=3x \Rightarrow x=7$.
- $AB=x+8=15$ cm, $AC=x+6=13$ cm.

Answer : AB = 15 cm, AC = 13 cm

Q13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.



Solution :

- Let ABCD circumscribe a circle with centre O, touching AB,BC,CD,DA at P,Q,R,S.
- Join O to the vertices and to the points of contact. Tangents from a vertex subtend equal angles at O:
- $\angle AOP=\angle AOS=a$, $\angle BOP=\angle BOQ=b$, $\angle COQ=\angle COR=c$, $\angle DOR=\angle DOS=d$.
- All eight angles around O add to 360°: $2(a+b+c+d)=360^\circ \Rightarrow a+b+c+d=180^\circ$.
- $\angle AOB+\angle COD=(a+b)+(c+d)=180^\circ$; similarly $\angle BOC+\angle AOD=180^\circ$.
- Hence opposite sides subtend supplementary angles at the centre. **(Proved)**

Concept Check — Test Your Understanding

Try each one in your own words first, then check the answer below.

Q1. Why is a tangent always perpendicular to the radius at the point of contact?

Answer: The radius OP to the point of contact is the shortest distance from the centre to the tangent line, and the shortest distance from a point to a line is the perpendicular distance. So $OP \perp$ tangent (Theorem 10.1).

Q2. From a point outside a circle, how many tangents can be drawn and how do their lengths compare?

Answer: Exactly two tangents can be drawn, and their lengths are equal (Theorem 10.2). This is proved by RHS congruence of the two right triangles formed.

Q3. Can a tangent be drawn from a point inside the circle? Why or why not?

Answer: No. Every line through an interior point meets the circle at two points, so it is a secant. A tangent needs the point to be on or outside the circle.

Q4. PQ is a tangent at P to a circle of centre O and radius 5 cm. If $OQ=13$ cm, why is $\triangle OPQ$ right-angled, and find PQ.

Answer: Since $OP \perp PQ$ (tangent $\perp$ radius), the right angle is at P. By Pythagoras, $PQ=\sqrt{(OQ^2-OP^2)}=\sqrt{(13^2-5^2)}=\sqrt{(144)}=12$ cm.

Q5. Two tangents from an external point make an angle of 80° between them. What is the angle $\angle AOB$ at the centre?

Answer: By Theorem 10.10 the two angles are supplementary: $\angle AOB=180^\circ-80^\circ=100^\circ$.

Q6. Which congruence rule proves that two tangents from an external point are equal, and why does it apply?

Answer: RHS. In $\triangle OAP$ and $\triangle OBP$: right angles at A,B (tangent $\perp$ radius), common hypotenuse OP, and equal sides $OA=OB$ (radii). So the triangles are congruent and $PA=PB$.

Q7. The angle between a tangent and a chord at the point of contact is 35°. What is the angle in the alternate segment?

Answer: By the Alternate Segment Theorem (Theorem 10.11) it is equal: the angle in the alternate segment is also 35°.

Concept Check — Test Your Understanding

Try each one in your own words first, then check the answer below.

Q8. Why are the tangents drawn at the two ends of a diameter parallel?

Answer: *Each tangent is perpendicular to the radius at its point of contact, and both radii lie along the same line (the diameter AB). Two lines perpendicular to the same line are parallel, so the tangents are parallel (Theorem 10.9).*