KUMAR CLASSES

Excellence in Mathematics

CLASS 10 MATHEMATICS

Chapter 2 : POLYNOMIALS – NCERT SOLUTIONS

Complete Concept Notes

Full Concepts • Solved Examples • Board Questions • Concept Check

Strong Concepts + Regular Practice = High Score

Exercise 2.1

Q1. The graphs of $y = p(x)$ are given in Fig. 2.10 below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$ in each case.

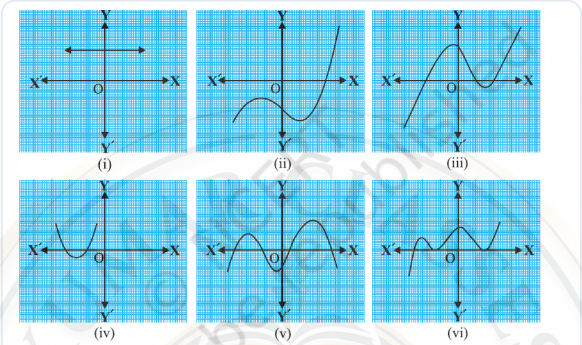

Fig. 2.10

Solution :

- The number of zeroes of $p(x)$ = the number of points where the graph of $y = p(x)$ meets (cuts or touches) the x-axis.
- **(i)** The graph does not meet the x-axis $\Rightarrow$ **0** zeroes.
- **(ii)** The graph meets the x-axis at **1** point $\Rightarrow$ **1** zero.
- **(iii)** The graph meets the x-axis at **3** points $\Rightarrow$ **3** zeroes.
- **(iv)** The graph meets the x-axis at **2** points $\Rightarrow$ **2** zeroes.
- **(v)** The graph meets the x-axis at **4** points $\Rightarrow$ **4** zeroes.
- **(vi)** The graph meets the x-axis at **3** points $\Rightarrow$ **3** zeroes.

Answer : (i) 0 (ii) 1 (iii) 3 (iv) 2 (v) 4 (vi) 3

Exercise 2.2

Q1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

Solution :

- **(i)** $x^2 - 2x - 8 = (x-4)(x+2)$. Zeroes: $x = 4,\ x = -2$.
- Verify: sum $= 4 + (-2) = 2 = -\frac{b}{a} = -\frac{-2}{1}$; product $= 4 \times (-2) = -8 = \frac{c}{a} = \frac{-8}{1}$.
- **(ii)** $4s^2 - 4s + 1 = (2s-1)^2$. Zeroes: $s = \frac{1}{2},\ \frac{1}{2}$.
- Verify: sum $= \frac{1}{2} + \frac{1}{2} = 1 = -\frac{-4}{4}$; product $= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = \frac{1}{4}$.
- **(iii)** $6x^2 - 7x - 3 = (3x+1)(2x-3)$. Zeroes: $x = -\frac{1}{3},\ \frac{3}{2}$.
- Verify: sum $= -\frac{1}{3} + \frac{3}{2} = \frac{7}{6} = -\frac{-7}{6}$; product $= -\frac{1}{3} \cdot \frac{3}{2} = -\frac{1}{2} = \frac{-3}{6}$.
- **(iv)** $4u^2 + 8u = 4u(u+2)$. Zeroes: $u = 0,\ -2$.
- Verify: sum $= 0 + (-2) = -2 = -\frac{8}{4}$; product $= 0 \times (-2) = 0 = \frac{0}{4}$.
- **(v)** $t^2 - 15 = (t - \sqrt{15})(t + \sqrt{15})$. Zeroes: $t = \sqrt{15},\ -\sqrt{15}$.
- Verify: sum $= 0 = -\frac{0}{1}$; product $= -15 = \frac{-15}{1}$.
- **(vi)** $3x^2 - x - 4 = (3x-4)(x+1)$. Zeroes: $x = \frac{4}{3},\ -1$.
- Verify: sum $= \frac{4}{3} + (-1) = \frac{1}{3} = -\frac{-1}{3}$; product $= \frac{4}{3} \times (-1) = -\frac{4}{3} = \frac{-4}{3}$.

Q2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

Solution :

TRICK *Quadratic polynomial* $= k\left[x^2 - (\text{Sum of zeroes})\,x + (\text{Product of zeroes})\right]$, *where* k *is any non-zero constant.*

- **(i)** $S = \frac{1}{4},\ P = -1 \Rightarrow k\left[x^2 - \frac{1}{4}x - 1\right]$; *taking* $k = 4:\ \mathbf{4x^2 - x - 4}$.
- **(ii)** $S = \sqrt{2},\ P = \frac{1}{3} \Rightarrow k\left[x^2 - \sqrt{2}\,x + \frac{1}{3}\right]$; *taking* $k = 3:\ \mathbf{3x^2 - 3\sqrt{2}\,x + 1}$.
- **(iii)** $S = 0,\ P = \sqrt{5} \Rightarrow k\left[x^2 - 0 \cdot x + \sqrt{5}\right]$; *taking* $k = 1:\ \mathbf{x^2 + \sqrt{5}}$.
- **(iv)** $S = 1,\ P = 1 \Rightarrow k\left[x^2 - x + 1\right]$; *taking* $k = 1:\ \mathbf{x^2 - x + 1}$.
- **(v)** $S = -\frac{1}{4},\ P = \frac{1}{4} \Rightarrow k\left[x^2 + \frac{1}{4}x + \frac{1}{4}\right]$; *taking* $k = 4:\ \mathbf{4x^2 + x + 1}$.
- **(vi)** $S = 4,\ P = 1 \Rightarrow k\left[x^2 - 4x + 1\right]$; *taking* $k = 1:\ \mathbf{x^2 - 4x + 1}$.

Concept Check — Test Your Understanding

Try each one in your own words first, then check the answer below.

Q1. What do the zeroes of a polynomial p(x) represent on its graph?

Answer: They are the x-coordinates of the points where the graph of $y=p(x)$ meets the x-axis.

Q2. What is the maximum number of zeroes of a quadratic polynomial and of a cubic polynomial?

Answer: A quadratic has at most 2 zeroes and a cubic has at most 3 zeroes.

Q3. If alpha,beta are the zeroes of ax^2+bx+c, what are alpha+beta and alphabeta?

Answer: alpha+beta=-b/a and alphabeta=c/a.

Q4. Can a quadratic polynomial have no real zeroes? What does its graph look like?

Answer: Yes. Its graph is a parabola that does not meet the x-axis at all.

Q5. If the sum and product of the zeroes are S and P, write a quadratic polynomial having them.

Answer: $k(x^2-Sx+P)$ for any non-zero constant k.

Q6. If a graph touches the x-axis at exactly one point, what does that say about the zeroes?

Answer: The polynomial has a repeated (equal) zero at that point.

Q7. If 0 is a zero of ax^2+bx+c, what must be true about c?

Answer: c=0, since the product of the zeroes =c/a would be 0.

Q8. A polynomial's graph meets the x-axis at 3 points. What is the least possible degree?

Answer: At least 3 (a cubic), since a polynomial of degree n has at most n zeroes.