✦ COMPLETE CONCEPT NOTES ✦

CLASS 12 • MATHEMATICS

VECTOR ALGEBRA

Chapter 10 • Complete Concept Notes

Basics
Directed segments, position vector, types of vectors

Algebra
Addition laws, scalar multiple, section formula

Components
Components, direction cosines & ratios

Dot Product
Scalar product, angle, projection

Cross Product
Vector product, area, unit normal

Formula Sheet
Quick revision + smart recall hooks

Strong Concepts + Regular Practice = High Score

Full Concepts · Verified Formulas · Clear Diagrams · Summary Tables

✦ INTRODUCTION — SCALAR & VECTOR ✦

1. INTRODUCTION — SCALAR & VECTOR

Only a number is enough?

- **Q.** *What is your height?* **A.** *178 cm.*
- *Needs* ***magnitude only.***
- ⇒ *This is a* **SCALAR**

Number + direction needed?

- **Q.** *How will you pass the football to your teammate?*
- **A.** *Kick it with enough power (magnitude)* ***towards*** *the teammate (direction).*
- ⇒ *This is a* **VECTOR**

Scalar (अदिश)

Has ***magnitude only.***

e.g. length, mass, time, distance, speed, area, volume, temperature, work, money, density.

Vector (सदिश)

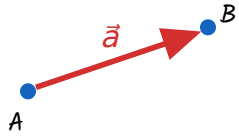
Has ***magnitude and direction.***

e.g. displacement, velocity, acceleration, force, weight, momentum, electric field.

2. LINE → DIRECTED LINE → DIRECTED LINE SEGMENT

Line *— no direction, no fixed ends.*

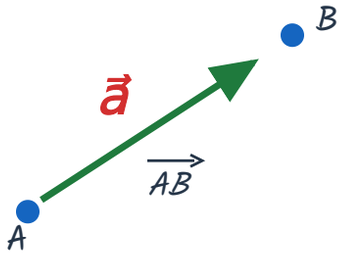
Directed line *— has direction.*



Directed line segment $\overrightarrow{AB}$ *—* ***a vector.***

✦ BASIC CONCEPTS (Part - I) ✦

1. DIRECTED LINE SEGMENT



Directed line segment AB = Vector AB = Vector $\vec{a}$ $\overrightarrow{AB}$ $\vec{a}$

→ Arrow = Direction

● Initial point of $\overrightarrow{AB}$ = A

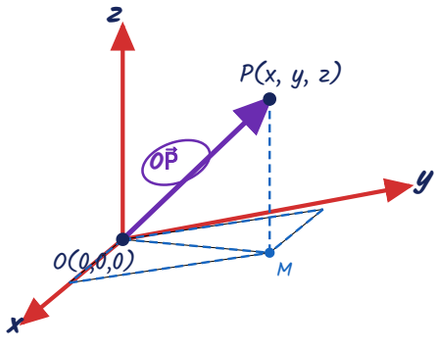
● Terminal point of $\overrightarrow{AB}$ = B

● Distance between initial & terminal points $\vec{a} = (\overrightarrow{AB})$

$$|\overrightarrow{AB}| = |\vec{a}|$$

= Magnitude (length) of the vector

2. POSITION VECTOR OF A POINT (स्थिति सदिश)



$\overrightarrow{OP}$ = Position vector of point P (w.r.t. origin O always)

Component form: $\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$

where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along x, y, z axes.

$$|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}$$

Note

★ If $P(x, 0, 0)$ then $\overrightarrow{OP} = x\hat{i}$

★ If $P(0, y, 0)$ then $\overrightarrow{OP} = y\hat{j}$

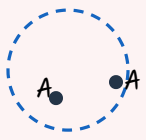
★ If $P(0, 0, z)$ then $\overrightarrow{OP} = z\hat{k}$

✦ TYPES OF VECTORS & SUMMARY ✦

★ 3. TYPES OF VECTORS ★

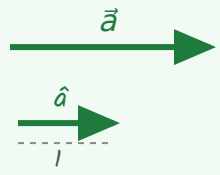
(i) ZERO VECTOR (Null Vector) $(\vec{0})$

- *Same initial & terminal points.*
- $|\vec{0}|$ = *Magnitude = 0*
- *Direction = Not defined (any direction)*



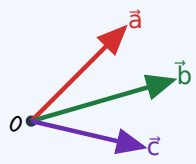
(ii) UNIT VECTOR

- *Magnitude = 1 unit*
- *Unit vector along* $\vec{a}$ *is* $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$
- $\hat{i}, \hat{j}, \hat{k}$ = *unit vectors along axes*



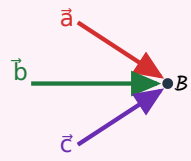
(iii) INITIAL (COINITIAL) VECTORS

- *Two or more vectors with the same initial point.*



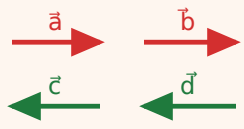
(iv) TERMINAL (COTERMINAL) VECTORS

- *Two or more vectors with the same terminal point.*



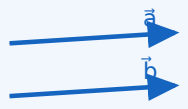
(v) COLLINEAR VECTORS

- *Vectors which are parallel to the same line.*
  - *Magnitude = same or different*
  - *Direction = same or opposite*



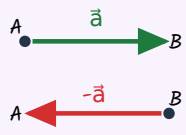
(vi) EQUAL VECTORS

- $\vec{a} = \vec{b}$ *if magnitude = same and direction = same.*

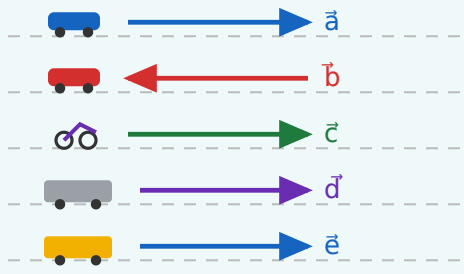


(vii) NEGATIVE OF A VECTOR

- *Magnitude = same, Direction = opposite*
- *For vector* $\vec{a} = \overrightarrow{AB}$, *Negative vector* $-\vec{a} = \overrightarrow{BA}$



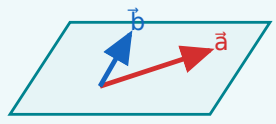
(viii) HIGHWAY (PARALLEL VECTORS)



Highway vectors are parallel (collinear) vectors along the same or parallel paths (like vehicles on a highway).

(ix) COPLANAR VECTORS

- *Two or more vectors lying in the* ***same plane***.
- *Any* ***two*** *vectors are always coplanar;* ***three or more*** *may or may not be.*



★ SUMMARY OF TYPES OF VECTORS ★

Type of Vector	Definition	Key Point
1 Zero Vector $(\vec{0})$	Same initial & terminal points; $\|\vec{0}\| = 0$, direction not defined.	Adding it changes nothing: $\vec{a} + \vec{0} = \vec{a}$.
2 Unit Vector $(\hat{a})$	Magnitude = 1 unit.	$\hat{a} = \dfrac{\vec{a}}{\|\vec{a}\|}$; e.g. $\hat{i}, \hat{j}, \hat{k}$.
3 Coinitial Vectors	Two or more with the same initial point.	Share the tail (start).
4 Coterminal Vectors	Two or more with the same terminal point.	Share the head (end).
5 Collinear Vectors	Parallel to the same line.	$\vec{a} = \lambda\vec{b}$ for some scalar λ.
6 Equal Vectors	Same magnitude and same direction.	$\vec{a} = \vec{b}$.
7 Negative of a Vector	Same magnitude, opposite direction.	$-\vec{a} = \overrightarrow{BA}$; $-(-\vec{a}) = \vec{a}$.
8 Coplanar Vectors	Lie in the same plane.	Any two vectors are always coplanar.

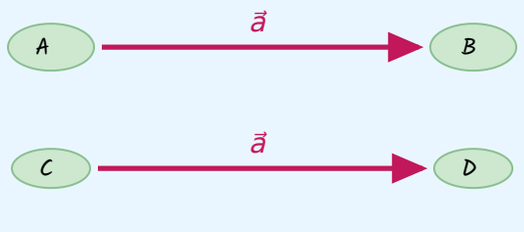
★ SMART RECALL ★

- ***Equal*** → *same size + same direction*
- ***Negative*** → *same size, opposite direction*
- ***Collinear*** → *parallel to one line*
- ***Coinitial*** → *share the tail (start)*
- ***Coterminal*** → *share the head (end)*
- ***Unit*** → *length 1;* ***Zero*** → *length 0, no direction*

FREE VECTORS

1. FREE VECTORS

- A vector is called a **free vector** if it can be shifted from one position to another **without changing its magnitude and direction**.
- **Graphical example:** the displacement of a boat from A to B is $\vec{a}$; the same displacement from C to D is also $\vec{a}$. Hence $\overrightarrow{AB} = \overrightarrow{CD}$.



2. SCALAR & VECTOR QUANTITIES

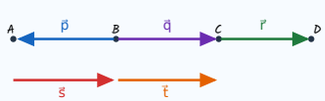
Scalars (magnitude only)

- Distance, Speed, Mass
- Time, Temperature
- Work, Energy, Volume

Vectors (magnitude + direction)

- Displacement, Velocity
- Acceleration, Force
- Momentum, Weight, Electric field

3. ANSWER THE FOLLOWING (DIAGRAM BASED)



In the figure, name the **1.** Collinear **2.** Equal **3.** Coinitial vectors.

SOLUTION

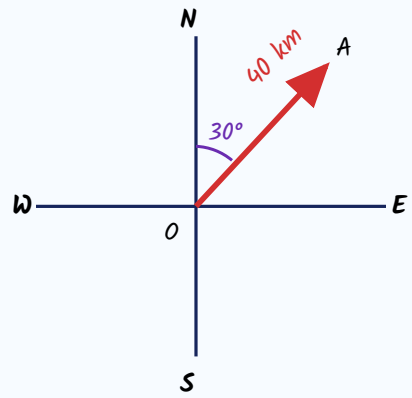
- **Collinear:** $\vec{p}, \vec{q}, \vec{r}, \vec{s}, \vec{t}$ — all lie along the same line.
- **Equal:** $\vec{q} = \vec{t}$ — same magnitude & same direction.
- **Coinitial:** $\vec{p}, \vec{q}$ — both start from point B.

✦ EXERCISE 10.1 ✦

1 Graphically represent a displacement of **40 km, 30° east of north**.

SOLUTION

- Draw a line OX towards **East**.
- At O, draw ON perpendicular to OX towards **North**.
- From O, draw a line making **30°** with ON towards East.
- Mark OA = **40 km** on it.
- Then $\overrightarrow{OA}$ represents the required displacement.



2 Classify each as a **scalar** or **vector**: (i) 10 kg (ii) 2 m north-west (iii) 40° (iv) 40 watt (v) 10^{-19} coulomb (vi) 20 m/s^2.

SOLUTION

Quantity	(i) 10 kg	(ii) 2 m N-W	(iii) 40°	(iv) 40 watt	(v) 10^{-19} C	(vi) 20 m/s^2
Type	Scalar	Vector	Scalar	Scalar	Scalar	Vector
Reason	magnitude only	magnitude + direction	magnitude only	magnitude only	magnitude only	magnitude + direction

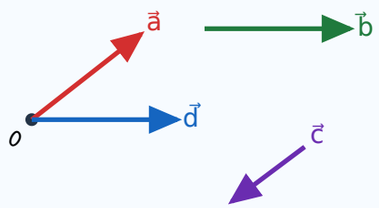
EXERCISE 10.1 (CONTD.)

3 Classify each as a **scalar** or **vector**: (i) time period (ii) distance (iii) force (iv) velocity (v) work done.

SOLUTION

Quantity	(i) Time period	(ii) Distance	(iii) Force	(iv) Velocity	(v) Work done
Type	Scalar	Scalar	Vector	Vector	Scalar
Reason	magnitude only	magnitude only	magnitude + direction	magnitude + direction	magnitude only

4 In the figure, identify: (a) Coinitial vectors (b) Equal vectors (c) Collinear but not equal vectors.



SOLUTION

- **(a) Coinitial:** $\vec{a}$ and $\vec{d}$ — same initial point O.
- **(b) Equal:** $\vec{b}$ and $\vec{d}$ — same magnitude & direction.
- **(c) Collinear but not equal:** $\vec{a}$ and $\vec{c}$ — parallel to one line but opposite/unequal.

5 Answer **True** or **False**:

SOLUTION

- (i) $\vec{a}$ and $-\vec{a}$ are collinear. **TRUE** (parallel to the same line).
- (ii) Two collinear vectors are always equal in magnitude. **FALSE**
- (iii) Two vectors having the same magnitude are collinear. **FALSE**
- (iv) Two collinear vectors having the same magnitude are equal. **FALSE** (may be opposite in direction).