

**MID-TERM EXAMINATION — Class 10 Mathematics****Chapters:** Real Numbers, Polynomials, Pair of Linear Equations, Triangles, Coordinate Geometry, Circles, Areas Related to Circles, Introduction to & Applications of Trigonometry**Max Marks: 80****Name:** _____**Time: 2 Hours****General Instructions:** All questions are compulsory. The paper has five sections — A (20×1 mark), B (5×2 marks), C (6×3 marks), D (4×5 marks) and E (3×4 marks, case-studies). Draw neat figures wherever required. Use of a calculator is **not** allowed. Take $\pi = \frac{22}{7}$ unless stated otherwise.**SECTION A**

Multiple Choice / Assertion-Reason — 1 mark each

- Q1.** If two positive integers are $a = x^3y^2$ and $b = xy^3$ (where x, y are prime numbers), then $\text{LCM}(a, b)$ is: [1]
- (a) x^3y^3
(b) xy^2
(c) x^3y^2
(d) x^2y^2
- Q2.** The greatest number which divides 70 and 125, leaving remainders 5 and 8 respectively, is: [1]
- (a) 13
(b) 65
(c) 875
(d) 1750
- Q3.** If one zero of $2x^2 - 3x + k$ is the reciprocal of the other, then the value of k is: [1]
- (a) 1
(b) 2
(c) -2
(d) 3
- Q4.** The quadratic polynomial whose zeroes are 3 and -2 is: [1]
- (a) $x^2 - x - 6$
(b) $x^2 + x - 6$
(c) $x^2 - x + 6$
(d) $x^2 + 5x + 6$
- Q5.** The pair of equations $3x + 2y = 5$ and $6x + 4y = k$ has infinitely many solutions when k equals: [1]
- (a) 5
(b) 10
(c) 15
(d) 20
- Q6.** If a pair of linear equations is consistent, then the two lines will be: [1]
- (a) always parallel
(b) always coincident
(c) intersecting or coincident
(d) none of these
- Q7.** The point on the x -axis which is equidistant from $(2, -5)$ and $(-2, 9)$ is: [1]
- (a) $(-7, 0)$
(b) $(7, 0)$
(c) $(0, -7)$

(d) (0, 7)

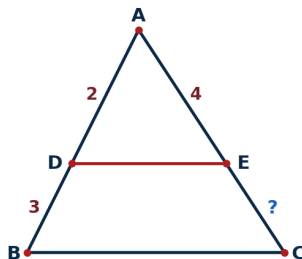
Q8. The distance of the point $(-6, 8)$ from the origin is:

[1]

- (a) 8
- (b) 10
- (c) 14
- (d) 6

Q9. In the given figure, $DE \parallel BC$ with $AD = 2$ cm, $DB = 3$ cm and $AE = 4$ cm. Then EC equals:

[1]



- (a) 5 cm
- (b) 6 cm
- (c) 7 cm
- (d) 8 cm

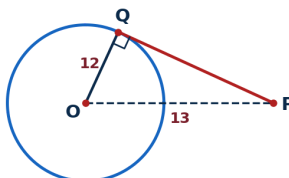
Q10. The triangle with sides 7 cm, 24 cm and 25 cm is:

[1]

- (a) acute-angled
- (b) right-angled
- (c) obtuse-angled
- (d) equilateral

Q11. In the given figure, PQ is a tangent from an external point P with $OP = 13$ cm and radius $OQ = 12$ cm. Then PQ equals:

[1]



- (a) 5 cm
- (b) 7 cm
- (c) $\sqrt{313}$ cm
- (d) 1 cm

Q12. The number of tangents that can be drawn to a circle from a point lying outside it is:

[1]

- (a) 1
- (b) 2
- (c) 3
- (d) infinitely many

Q13. The value of $\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$ is:

[1]

- (a) 0
- (b) 1
- (c) $\frac{1}{2}$
- (d) $\sqrt{3}$

Q14. If $\sin A = \frac{1}{2}$, then the value of $3 \cos A - 4 \cos^3 A$ is:

[1]

- (a) 0

- (b) 1
- (c) -1
- (d) $\frac{1}{2}$

Q15. The expression $(1 + \tan^2 \theta)$ is equal to: [1]

- (a) $\csc^2 \theta$
- (b) $\sec^2 \theta$
- (c) $\cot^2 \theta$
- (d) 1

Q16. The angle of elevation of the top of a tower from a point 100 m away from its foot is 30° . The height of the tower is: [1]

- (a) $\frac{100}{\sqrt{3}}$ m
- (b) $100\sqrt{3}$ m
- (c) 50 m
- (d) 100 m

Q17. If the circumference and the area of a circle are numerically equal, then the radius of the circle is: [1]

- (a) 1
- (b) 2
- (c) 4
- (d) π

Q18. The area of a sector of a circle of radius 6 cm and central angle 60° is: [1]

- (a) $6\pi \text{ cm}^2$
- (b) $3\pi \text{ cm}^2$
- (c) $12\pi \text{ cm}^2$
- (d) $36\pi \text{ cm}^2$

Q19. Assertion (A): The HCF of 12 and 15 is 3. [1]

Reason (R): The HCF of two numbers is the largest number that divides both of them exactly.

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

Q20. Assertion (A): The point (0, 4) lies on the y -axis. [1]

Reason (R): The x -coordinate of every point on the y -axis is 0.

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

SECTION B

Very Short Answer — 2 marks each

Q21. Find the largest number that divides 245 and 1029, leaving remainder 5 in each case. [2]

Q22. If α, β are the zeroes of $x^2 - 6x + 8$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$. [2]

Q23. For what value of k does the pair $kx + 3y = k - 3$ and $12x + ky = k$ have infinitely many solutions? [2]

Q24. In $\triangle ABC$, $DE \parallel BC$ with $\frac{AD}{DB} = \frac{3}{5}$ and $AC = 5.6$ cm. Find the length of AE . [2]

Q25. Prove that $(1 + \cot A - \csc A)(1 + \tan A + \sec A) = 2$.

[2]

SECTION C

Short Answer — 3 marks each

Q26. Prove that $\sqrt{3}$ is an irrational number.

[3]

Q27. Find the zeroes of the polynomial $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ and verify the relationship between the zeroes and the coefficients.

[3]

Q28. A boat covers 30 km upstream and 44 km downstream in 10 hours; it covers 40 km upstream and 55 km downstream in 13 hours. Find the speed of the boat in still water and the speed of the stream.

[3]

Q29. Prove that a parallelogram circumscribing a circle is a rhombus.

[3]

Q30. Find the area of the triangle whose vertices are $(1, -1)$, $(-4, 6)$ and $(-3, -5)$.

[3]

Q31. Prove that $(\sin A + \csc A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$.

[3]

SECTION D

Long Answer — 5 marks each

Q32. **State and prove the Basic Proportionality Theorem (Thales' Theorem):** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

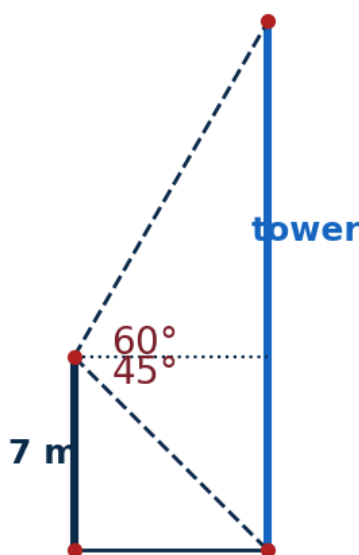
[5]

Q33. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

[5]

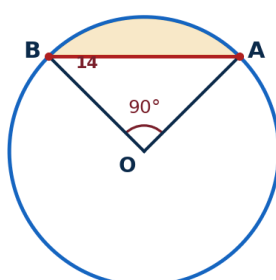
Q34. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° . Find the height of the tower. (Use $\sqrt{3} = 1.73$)

[5]

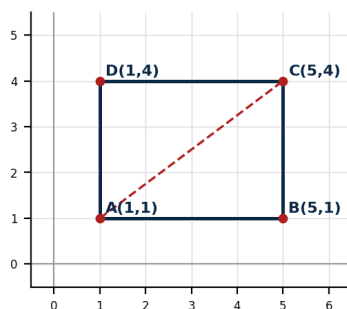


Q35. A chord of a circle of radius 14 cm subtends a right angle at the centre. Find the area of the corresponding minor segment and the major segment. (Use $\pi = \frac{22}{7}$)

[5]

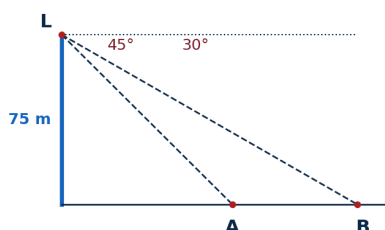


- Q36. Rangoli design on a grid.** During a school function, four students stand at the corners of a rangoli marked on a coordinate grid at $A(1, 1)$, $B(5, 1)$, $C(5, 4)$ and $D(1, 4)$. [4]



- (i) Find the distance AB . [1]
 (ii) Find the distance BC . [1]
 (iii) Find the length of the diagonal AC , and state the type of quadrilateral $ABCD$. [2]

- Q37. Lighthouse and ships.** From the top L of a lighthouse 75 m high, the angles of depression of two ships A and B on the same side, in a straight line with the base, are 45° and 30° respectively. [4]



- (i) Find the distance of ship A from the base of the lighthouse. [1]
 (ii) Find the distance of ship B from the base. [1]
 (iii) Find the distance between the two ships. (Use $\sqrt{3} = 1.73$) [2]

- Q38. Wall clock.** The circular face of a wall clock has radius 14 cm; the minute hand sweeps out a sector as time passes. [4]

- (i) Find the circumference of the clock face. [1]
 (ii) Find the total area of the clock face. [1]
 (iii) Find the area swept by the minute hand in 10 minutes (a 60° sector). [2]

— All the Best! —

ANSWER KEY & MARKING SCHEME

SECTION A

A — Answers

- Q1. (a) x^3y^3
- Q2. (a) 13 [HCF of $70 - 5 = 65$ and $125 - 8 = 117$ is 13]
- Q3. (b) 2 [product of zeroes = $k/2 = 1$]
- Q4. (a) $x^2 - x - 6$
- Q5. (b) 10
- Q6. (c) intersecting or coincident
- Q7. (a) $(-7, 0)$
- Q8. (b) 10
- Q9. (b) 6 cm
- Q10. (b) right-angled [$7^2 + 24^2 = 25^2$]
- Q11. (a) 5 cm
- Q12. (b) 2
- Q13. (b) 1 [= $\sin 90^\circ$]
- Q14. (a) 0 [= $-\cos 3A = -\cos 90^\circ$]
- Q15. (b) $\sec^2 \theta$
- Q16. (a) $\frac{100}{\sqrt{3}}$ m
- Q17. (b) 2
- Q18. (a) $6\pi \text{ cm}^2$
- Q19. (a) Both A and R true, R correct explanation
- Q20. (a) Both A and R true, R correct explanation

SECTION B

B — Answers

- Q21. $245 - 5 = 240$, $1029 - 5 = 1024$. HCF(240, 1024) = 16.
- Q22. $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{6}{8} = \frac{3}{4}$.
- Q23. For infinitely many solutions $\frac{k}{12} = \frac{3}{k} = \frac{k-3}{k}$. From $\frac{k}{12} = \frac{3}{k} \Rightarrow k^2 = 36$; consistency with the third ratio gives $k = 6$.
- Q24. $\frac{AE}{EC} = \frac{AD}{DB} = \frac{3}{5}$, so $AE = \frac{3}{8} \times 5.6 = 2.1$ cm.
- Q25. Expand: $= 1 + \tan A + \sec A + \cot A + 1 + \csc A - \csc A - \sec A - \csc A \cot A$. Using $\cot A \csc A = \frac{\cos A}{\sin^2 A}$ and simplifying $\Rightarrow 2$.
(Verified by identity.)

SECTION C

C — Answers

- Q26. Assume $\sqrt{3} = \frac{p}{q}$ in lowest terms $\Rightarrow 3q^2 = p^2 \Rightarrow 3 \mid p$; put $p = 3m \Rightarrow q^2 = 3m^2 \Rightarrow 3 \mid q$. So 3 divides both p, q — contradiction.
Hence $\sqrt{3}$ is irrational.
- Q27. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = (4x - \sqrt{3})(\sqrt{3}x + 2)$... solving, zeroes = $\frac{\sqrt{3}}{4}, -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$. Sum = $-\frac{5}{4\sqrt{3}} = -\frac{b}{a}$; Product = $-\frac{1}{2} = \frac{c}{a}$.
Verified.
- Q28. Let $u = \frac{1}{b-s}, v = \frac{1}{b+s}$: $30u + 44v = 10, 40u + 55v = 13 \Rightarrow u = \frac{1}{5}, v = \frac{1}{11}$. So $b-s = 5, b+s = 11 \Rightarrow$ boat = 8 km/h, stream = 3 km/h.
- Q29. Tangents from an external point are equal: $AP = AS, BP = BQ, CR = CQ, DR = DS$. Adding, $AB + CD = AD + BC$. In a parallelogram $AB = CD, BC = AD$, so $2AB = 2BC \Rightarrow AB = BC$. Equal adjacent sides $\Rightarrow$ rhombus.

Q30. Area = $\frac{1}{2}|1(6+5) + (-4)(-5+1) + (-3)(-1-6)| = \frac{1}{2}|11+16+21| = 24$ sq. units.

Q31. $(\sin A + \csc A)^2 + (\cos A + \sec A)^2 = \sin^2 A + \cos^2 A + \csc^2 A + \sec^2 A + 4 = 1 + (1 + \cot^2 A) + (1 + \tan^2 A) + 4 = 7 + \tan^2 A + \cot^2 A$.
Proved.

SECTION D

D — Answers

Q32. Given $\triangle ABC$ with $DE \parallel BC$. Join BE, CD ; draw $DM \perp AC, EN \perp AB$. Then $\frac{\ar(ADE)}{\ar(BDE)} = \frac{AD}{DB}$ and $\frac{\ar(ADE)}{\ar(CDE)} = \frac{AE}{EC}$. Since $DE \parallel BC$, $\ar(BDE) = \ar(CDE)$ (same base DE , same parallels). Hence $\frac{AD}{DB} = \frac{AE}{EC}$.

Q33. $\frac{24}{18-s} - \frac{24}{18+s} = 1 \Rightarrow 48s = 324 - s^2 \Rightarrow s^2 + 48s - 324 = 0 \Rightarrow (s-6)(s+54) = 0 \Rightarrow s = 6$. Speed of stream = 6 km/h.

Q34. Let horizontal distance = d . From depression 45° to foot: $\tan 45^\circ = \frac{7}{d} \Rightarrow d = 7$. From elevation 60° : extra height = $d \tan 60^\circ = 7\sqrt{3}$. Tower height = $7 + 7\sqrt{3} = 7(1 + \sqrt{3}) = 7(2.73) = 19.11$ m.

Q35. Sector $(90^\circ) = \frac{1}{4} \times \frac{22}{7} \times 196 = 154$ cm²; $\triangle = \frac{1}{2} \times 14 \times 14 = 98$ cm². Minor segment = $154 - 98 = 56$ cm². Major segment = $\pi r^2 - 56 = 616 - 56 = 560$ cm².

SECTION E

E — Answers

Q36. (i) $AB = \sqrt{5-1}^2 + 0 = 4$ units.
(ii) $BC = \sqrt{0^2 + (4-1)^2} = 3$ units.
(iii) $AC = \sqrt{4^2 + 3^2} = 5$ units; since $AB = CD = 4, BC = DA = 3$ with right angles, $ABCD$ is a rectangle.

Q37. (i) $A: \tan 45^\circ = \frac{d_1}{75} \Rightarrow d_1 = 75$ m.
(ii) $B: \tan 30^\circ = \frac{d_2}{75} \Rightarrow d_2 = 75 \tan 30^\circ = 75 \times \frac{1}{\sqrt{3}} = 75 \times 0.577 = 43.27$ m.
(iii) $AB = d_1 - d_2 = 75 - 43.27 = 31.73$ m.

Q38. (i) Circumference = $2\pi r = 2 \times \frac{22}{7} \times 14 = 88$ cm.
(ii) Area = $\frac{1}{2} \times 22 \times 14 = 154$ cm².
(iii) 10° sector = $\frac{10}{360} \times \pi r^2 = \frac{10}{360} \times \frac{22}{7} \times 14^2 = 10.27$ cm².