

**SAMPLE PAPER — Class 12 Mathematics (NCERT Part I)**

Chapters: Relations & Functions, Inverse Trigonometric Functions, Matrices, Determinants, Continuity & Differentiability, Application of Derivatives

Max Marks: 80

Name: _____

Time: 2 Hours

General Instructions: All questions are compulsory. Five sections — A (20×1), B (5×2), C (6×3), D (4×5), E (3×4, case-studies). Draw neat figures where required. Take $\pi = \frac{22}{7}$ unless stated otherwise.

SECTION A

Multiple Choice / Assertion-Reason — 1 mark each

- Q1.** The number of reflexive relations on a set containing 3 elements is: [1]
(a) 2^9
(b) 2^6
(c) 2^3
(d) 512
- Q2.** The function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 3x - 4$ is: [1]
(a) one-one but not onto
(b) onto but not one-one
(c) a bijection
(d) neither one-one nor onto
- Q3.** The principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ is: [1]
(a) $\frac{\pi}{6}$
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$
(d) $\frac{5\pi}{6}$
- Q4.** The domain of $\cos^{-1} x$ is: [1]
(a) $\mathbb{R}$
(b) $[-1, 1]$
(c) $[0, \pi]$
(d) $(-1, 1)$
- Q5.** The value of $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$ is: [1]
(a) $\frac{2\pi}{3}$
(b) $\frac{\pi}{3}$
(c) $-\frac{\pi}{3}$
(d) $\frac{\pi}{6}$
- Q6.** If A is of order 3×4 and B is of order 4×2 , then the order of AB is: [1]

- (a) 3×2
- (b) 4×4
- (c) 2×3
- (d) not defined

Q7. The number of elements in a 3×3 matrix is: [1]

- (a) 3
- (b) 6
- (c) 9
- (d) 27

Q8. If $A = [a_{ij}]$ is a 2×2 matrix with $a_{ij} = i + j$, then a_{21} equals: [1]

- (a) 2
- (b) 3
- (c) 4
- (d) 1

Q9. If A is a square matrix of order 3 with $|A| = 5$, then $|2A|$ equals: [1]

- (a) 10
- (b) 40
- (c) 20
- (d) 80

Q10. The value of the determinant $\begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix}$ is: [1]

- (a) 2
- (b) 6
- (c) 10
- (d) -2

Q11. If A is a singular matrix, then $|A|$ equals: [1]

- (a) 1
- (b) 0
- (c) -1
- (d) undefined

Q12. The function $f(x) = |x|$ is: [1]

- (a) continuous and differentiable at $x = 0$
- (b) continuous but not differentiable at $x = 0$
- (c) neither continuous nor differentiable at $x = 0$
- (d) differentiable but not continuous at $x = 0$

Q13. $\frac{d}{dx}(\log x)$ equals: [1]

- (a) x
- (b) $\frac{1}{x}$
- (c) $\ln x$

(d) $-\frac{1}{x^2}$

- Q14.** $\frac{d}{dx}(e^{2x})$ equals: [1]
- (a) e^{2x}
(b) $2e^{2x}$
(c) $2xe^{2x}$
(d) e^2

- Q15.** The slope of the tangent to the curve $y = x^2$ at the point $(1, 1)$ is: [1]
- (a) 1
(b) 2
(c) 3
(d) 0

- Q16.** The function $f(x) = x^2$ is strictly increasing on: [1]
- (a) $(-\infty, 0)$
(b) $(0, \infty)$
(c) $\mathbb{R}$
(d) $(-\infty, \infty) \setminus \{0\}$

- Q17.** The value of $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$ is: [1]
- (a) $\frac{7\pi}{6}$
(b) $\frac{5\pi}{6}$
(c) $\frac{\pi}{6}$
(d) $-\frac{5\pi}{6}$

- Q18.** In a skew-symmetric matrix, all the diagonal elements are: [1]
- (a) 1
(b) 0
(c) equal
(d) positive

- Q19. Assertion (A):** For any square matrix A , the matrix $A + A'$ is symmetric. [1]
Reason (R): $(A + A')' = A + A'$.
- (a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

- Q20. Assertion (A):** The function $f(x) = x^3$ is strictly increasing on $\mathbb{R}$. [1]
Reason (R): $f'(x) = 3x^2 \geq 0$ for all $x \in \mathbb{R}$.
- (a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false

(d) A is false but R is true

SECTION B

Very Short Answer — 2 marks each

Q21. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$. [2]

Q22. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find $|A|$ and state whether A is invertible. [2]

Q23. Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2x + 3$ is one-one. [2]

Q24. Find $\frac{dy}{dx}$ if $y = \sin(x^2)$. [2]

Q25. Find the interval in which $f(x) = x^2 - 4x + 6$ is strictly increasing. [2]

SECTION C

Short Answer — 3 marks each

Q26. Show that the relation R on $A = \{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$ is reflexive and symmetric but examine whether it is transitive. [3]

Q27. Express the matrix $A = \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix. [3]

Q28. If $x^2 + y^2 = 25$, find $\frac{dy}{dx}$. [3]

Q29. Prove that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$. [3]

Q30. If $y = x^x$, find $\frac{dy}{dx}$. [3]

Q31. Verify Rolle's theorem for the function $f(x) = x^2 - 4$ on the interval $[-2, 2]$. [3]

SECTION D

Long Answer — 5 marks each

Q32. Using matrices, solve the system of equations: [5]
 $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$.

Q33. Find the intervals in which $f(x) = 2x^3 - 3x^2 - 36x + 7$ is increasing or decreasing, and find its points of local maximum and local minimum. [5]

Q34. A square sheet of tin of side 18 cm is to be made into an open box by cutting equal squares from each corner and folding up the flaps. Find the side of the square to be cut so that the volume of the box is maximum. [5]

Q35. [5]
Using properties of determinants, prove that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$.

SECTION E

Case-Study Based — 4 marks each

Q36. Profit analysis. The profit (in thousands of rupees) of a small firm on producing x hundred units is modelled by $P(x) = -3x^2 + 30x - 50$. [4]

(i) Find $P'(x)$. [1]

(ii) Find the production level x at which the profit is maximum. [2]

(iii) Find the maximum profit.

[1]

Q37. Triangular plot. A triangular plot has vertices $A(1, 0)$, $B(6, 0)$ and $C(4, 3)$ (units in metres). [4]

(i) Write the determinant expression for its area. [1]

(ii) Find the area of the plot. [2]

(iii) Are the points $(1, 0)$, $(6, 0)$ and $(3, 0)$ collinear? Justify. [1]

Q38. Continuous cost function. A courier charges follow $f(x) = \begin{cases} kx + 1, & x \leq 5 \\ 3x - 5, & x > 5 \end{cases}$ where x is the weight in kg. [4]

(i) Write the condition for f to be continuous at $x = 5$. [1]

(ii) Find the value of k . [2]

(iii) Find $f(5)$. [1]

— All the Best! —

ANSWER KEY & MARKING SCHEME

SECTION A

Answers

- Q1. (b) $2^6 (= 64)$
- Q2. (c) a bijection
- Q3. (a) $\frac{\pi}{6}$
- Q4. (b) $[-1, 1]$
- Q5. (b) $\frac{\pi}{3}$
- Q6. (a) 3×2
- Q7. (c) 9
- Q8. (b) 3 ($a_{21} = 2 + 1$)
- Q9. (b) 40 ($2^3 \times 5$)
- Q10. (a) 2
- Q11. (b) 0
- Q12. (b) continuous but not differentiable at $x = 0$
- Q13. (b) $\frac{1}{x}$
- Q14. (b) $2e^{2x}$
- Q15. (b) 2
- Q16. (b) $(0, \infty)$
- Q17. (b) $\frac{5\pi}{6}$
- Q18. (b) 0
- Q19. (a) Both true, R explains A
- Q20. (a) Both true, R explains A

SECTION B

Answers

- Q21. $\sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$, $\cos^{-1}(-\frac{1}{2}) = \frac{2\pi}{3}$. Sum = $-\frac{\pi}{6} + \frac{2\pi}{3} = \frac{\pi}{2}$.
- Q22. $|A| = 1 \times 4 - 2 \times 3 = -2 \neq 0$, so A is invertible.
- Q23. If $f(x_1) = f(x_2) \Rightarrow 2x_1 + 3 = 2x_2 + 3 \Rightarrow x_1 = x_2$. Hence f is one-one.
- Q24. $\frac{dy}{dx} = \cos(x^2) \cdot 2x = 2x \cos(x^2)$.
- Q25. $f'(x) = 2x - 4 > 0 \Rightarrow x > 2$. Increasing on $(2, \infty)$.

SECTION C

Answers

- Q26. Reflexive: $(1, 1), (2, 2), (3, 3) \in R$ ✓. Symmetric: $(1, 2) \in R$ and $(2, 1) \in R$ ✓. Transitive: $(2, 1)$ and $(1, 2)$ give $(2, 2) \in R$; $(1, 2), (2, 1)$ give $(1, 1) \in R$ — all required pairs present, so R is also transitive (in fact an equivalence relation).
- Q27. Misplaced &. Symmetric part Misplaced &; skew part Misplaced &; $A = P + Q$.
- Q28. Differentiating $x^2 + y^2 = 25$: $2x + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$.

Q29. $\tan^{-1}1 + \tan^{-1}2 = \pi + \tan^{-1}\frac{1+2}{1-2} = \pi - \tan^{-1}3$. Hence the sum $= \pi - \tan^{-1}3 + \tan^{-1}3 = \pi$.
Proved.

Q30. $\ln y = x \ln x \Rightarrow \frac{1}{y} \frac{dy}{dx} = \ln x + 1 \Rightarrow \frac{dy}{dx} = x^x(1 + \ln x)$.

Q31. f is continuous on $[-2, 2]$, differentiable on $(-2, 2)$, and $f(-2) = f(2) = 0$. So $\exists c$ with $f'(c) = 2c = 0 \Rightarrow c = 0 \in (-2, 2)$. Rolle's theorem verified.

SECTION D

Answers

Q32. $AX = B$ with $|A| = -1 \neq 0$; solving $X = A^{-1}B$ gives $x = 1, y = 2, z = 3$.

Q33. $f'(x) = 6x^2 - 6x - 36 = 6(x-3)(x+2)$. Increasing on $(-\infty, -2) \cup (3, \infty)$, decreasing on $(-2, 3)$. Local maximum at $x = -2$ ($f = 51$); local minimum at $x = 3$ ($f = -74$).

Q34. $V = x(18-2x)^2$. $\frac{dV}{dx} = 12(9-x)(3-x) = 0 \Rightarrow x = 3$ (rejecting $x = 9$). $V'' < 0$ at $x = 3$, so maximum. Side to cut = 3 cm (max volume = 432 cm³).

Q35. C_2 to $C_2 - C_1$, C_3 to $C_3 - C_1$ gives $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

SECTION E

Answers

Q36. (i) $P'(x) = -6x + 30$.

(ii) $P'(x) = 0 \Rightarrow x = 5$; $P''(x) = -6 < 0$, so maximum at $x = 5$ (hundred units).

(iii) $P(5) = -75 + 150 - 50 = 25$ (thousand rupees).

Q37. (i) Area = 7.5 m².

(ii) Area = $\frac{1}{2} |1(0-3) + 6(3-0) + 4(0-0)| = \frac{1}{2} |12| = 6$ m².

(iii) $(1, 0), (6, 0), (3, 0)$ all lie on the x -axis $\Rightarrow$ area = 0, so they are collinear.

Q38. (i) For continuity at $x = 5$: $\lim_{x \rightarrow 5^-} f = \lim_{x \rightarrow 5^+} f = f(5)$, i.e. $5k + 1 = 3(5) - 5$.

(ii) $5k + 1 = 10 \Rightarrow k = \frac{9}{5}$.

(iii) $f(5) = 5k + 1 = 10$.