

**MID-TERM EXAMINATION — Class 12 Mathematics**

Chapters: Relations & Functions, Inverse Trigonometric Functions, Matrices, Determinants, Continuity & Differentiability, Applications of Derivatives, Integrals, Vector Algebra, Three-Dimensional Geometry, Linear Programming

Max Marks:
80

Name: _____

Time: 2 Hours

General Instructions: All questions are compulsory. Five sections — A (20×1), B (5×2), C (6×3), D (4×5), E (3×4, case-studies). Draw neat figures where required. Take $\pi = \frac{22}{7}$ unless stated otherwise.

SECTION A

Multiple Choice / Assertion-Reason — 1 mark each

- Q1.** Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 2x + 3$. Then f is: [1]
- (a) one-one but not onto
(b) onto but not one-one
(c) one-one and onto
(d) neither one-one nor onto
- Q2.** If A is a square matrix of order 3 such that $|A| = 4$, then $|\text{adj } A|$ is equal to: [1]
- (a) 16
(b) 4
(c) 64
(d) 8
- Q3.** **Assertion (A):** The function $f(x) = |x|$ is continuous at $x = 0$ but not differentiable at $x = 0$. [1]
Reason (R): A function that is differentiable at a point must be continuous there, but continuity does not guarantee differentiability.
- (a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true
- Q4.** The value of $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$ is: [1]
- (a) $\frac{\pi}{3}$
(b) $\frac{2\pi}{3}$
(c) $-\frac{\pi}{3}$
(d) $\frac{\pi}{6}$
- Q5.** The direction cosines of a line equally inclined to all three coordinate axes are: [1]
- (a) (1, 1, 1)
(b) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$
(c) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

(d) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Q6. Assertion (A): The function $f(x) = x^3$ is strictly increasing on $\mathbb{R}$. [1]

Reason (R): If $f'(x) > 0$ for all x in an interval, then f is strictly increasing on that interval.

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

Q7. The value of $\int_{-1}^1 x^3 \cos x \, dx$ is: [1]

- (a) 0
- (b) 1
- (c) 2
- (d) -2

Q8. In a Linear Programming Problem, if the feasible region is bounded, the optimal value of the objective function is attained at: [1]

- (a) the centre of the feasible region
- (b) any arbitrary point of the feasible region
- (c) at least one corner point of the feasible region
- (d) the origin only

Q9. Assertion (A): If $\vec{a} \cdot \vec{b} = 0$, where $\vec{a}, \vec{b}$ are non-zero vectors, then $\vec{a}$ and $\vec{b}$ are perpendicular. [1]

Reason (R): The dot product of two non-zero vectors is zero if and only if the angle between them is 90° .

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

Q10. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to: [1]

- (a) A
- (b) I
- (c) $3A$
- (d) $I + A$

Q11. The angle between the lines with direction ratios $2, 1, -2$ and $1, 2, 2$ is: [1]

- (a) 0°
- (b) 30°
- (c) 60°
- (d) 90°

Q12. Assertion (A): For any square matrix A , the matrix $A + A^T$ is always symmetric. [1]

Reason (R): A square matrix M is said to be symmetric if $M^T = M$.

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is not the correct explanation of A
- (c) A is true but R is false

(d) A is false but R is true

Q13. If A is a square matrix of order 3 such that $|A| = 5$, then $|3A|$ is equal to: [1]

- (a) 15
- (b) 45
- (c) 135
- (d) 225

Q14. The interval in which $f(x) = x^3 - 3x^2 + 1$ is strictly decreasing is: [1]

- (a) $(0, 2)$
- (b) $(-\infty, 0)$
- (c) $(2, \infty)$
- (d) $(-\infty, \infty)$

Q15. Assertion (A): The lines with direction ratios $(1, 2, 3)$ and $(2, 4, 6)$ are parallel. [1]

Reason (R): Two lines are parallel if and only if their direction ratios are proportional.

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

Q16. The domain of $f(x) = \cos^{-1}(2x - 1)$ is: [1]

- (a) $(0, 1)$
- (b) $[0, 1]$
- (c) $[-1, 1]$
- (d) $(-1, 1)$

Q17. If A is a skew-symmetric matrix of order 3, then $|A|$ is equal to: [1]

- (a) 0
- (b) 1
- (c) -1
- (d) depends on the entries of A

Q18. Assertion (A): Rolle's theorem is applicable to $f(x) = |x|$ on $[-1, 1]$. [1]

Reason (R): Rolle's theorem states that if f is continuous on $[a, b]$, differentiable on (a, b) and $f(a) = f(b)$, then there exists $c \in (a, b)$ such that $f'(c) = 0$.

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is not the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

Q19. The value of $\int_0^{\pi/2} \sin^2 x \, dx$ is: [1]

- (a) $\frac{\pi}{4}$
- (b) $\frac{\pi}{2}$
- (c) π
- (d) 0

- Q20. Assertion (A):** The function $f(x) = x^2$ attains a local minimum at $x = 0$. [1]
Reason (R): At a point of local minimum, $f'(x) = 0$ and $f''(x) > 0$.
 (a) Both A and R are true and R is the correct explanation of A
 (b) Both A and R are true but R is not the correct explanation of A
 (c) A is true but R is false
 (d) A is false but R is true

SECTION B

Very Short Answer — 2 marks each

- Q21.** If $x = a(\theta - \sin \theta)$ and $y = a(1 - \cos \theta)$, find $\frac{dy}{dx}$. [2]
- Q22.** Using differentials, find the approximate value of $\sqrt{25.3}$. [2]
- Q23.** If $f(x) = \begin{cases} \frac{\sin 3x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, find the value of k . [2]
- Q24.** Find a unit vector perpendicular to both $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$. [2]
- Q25.** Find the projection of the vector $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$. [2]

SECTION C

Short Answer — 3 marks each

- Q26.** Verify Rolle's theorem for the function $f(x) = x^2 - 4x + 3$ on the interval $[1, 3]$. [3]
- Q27.** Using properties of determinants, prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$. [3]
- Q28.** Evaluate: $\int \frac{x}{(x-1)(x-2)} dx$. [3]
- Q29.** Find the shortest distance between the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$. [3]
- Q30.** Using elementary row operations, find the inverse of the matrix $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$. [3]
- Q31.** Evaluate: $\int \frac{dx}{x^2 - 6x + 13}$. [3]

SECTION D

Long Answer — 5 marks each

- Q32.** A company manufactures two products **A** and **B** on the same machine. Each unit of **A** requires 3 hours of machine time and 1 hour of labour, while each unit of **B** requires 1 hour of machine time and 2 hours of labour. The machine is available for at most 12 hours per day and labour for at most 10 hours per day. The profit is **Rs. 60** per unit of **A** and **Rs. 40** per unit of **B**. Formulate and solve the LPP to maximise the profit. [5]
- Q33.** Using the matrix (inverse) method, solve the system of equations: [5]
 $x + y + z = 6, \quad y + 3z = 11, \quad x - 2y + z = 0.$

Q34. Show that the height of a right circular cone of maximum volume that can be inscribed in a sphere of radius R is $\frac{4R}{3}$. Hence find the maximum volume of such a cone in terms of R . [5]

Q35. Find the vector and Cartesian equations of the plane passing through the points $A(1, 1, 1)$, $B(1, -1, 2)$ and $C(-2, -2, 2)$. Hence find the distance of the point $P(1, 2, 3)$ from this plane, and the angle this plane makes with the plane $2x - y + 2z + 5 = 0$. [5]

SECTION E

Case-Study Based – 4 marks each

Q36. Open box. An open rectangular box is made from a square metal sheet of side 18 cm by cutting equal squares of side x cm from each corner and folding up the sides. The box has a square base and vertical sides. [4]

(i) Express the volume $V(x)$ of the box as a function of x , and state the interval in which x must lie. [1]

(ii) Find $\frac{dV}{dx}$ and determine the critical point(s) of $V(x)$ in the valid interval. [1]

(iii) Using the second derivative test, show that the volume is maximum at this value of x , and find the maximum volume. [2]

Q37. Notebook sales. A shop's sales of three notebook types over two days are [4]

$Q = \begin{pmatrix} 50 & 30 & 20 \\ 40 & 35 & 25 \end{pmatrix}$ (rows: Day 1, Day 2; columns: Types A, B, C) and the selling prices (Rs.)

are $P = \begin{pmatrix} 10 \\ 15 \\ 20 \end{pmatrix}$.

(i) State the order of the product QP and why it is defined. [1]

(ii) Find the total revenue earned on Day 1 and on Day 2 using matrix multiplication. [2]

(iii) If the price of Type C is increased by Rs. 5 (others unchanged), find the new total revenue on Day 1. [1]

Q38. Motion of a ball. A ball moves along a straight track; its position (in metres) from a fixed point O after t seconds is $s(t) = t^3 - 6t^2 + 9t + 4$, $t \geq 0$. [4]

(i) Find the velocity as a function of t and its value at $t = 2$ s. [1]

(ii) Find the value(s) of t at which the ball is momentarily at rest. [1]

(iii) Find the acceleration at $t = 1$ s and the interval(s) in which the ball moves forward. [2]

— All the Best! —

ANSWER KEY & MARKING SCHEME

SECTION A

Answers

- Q1. (c) one-one and onto
- Q2. (a) 16 ($|\text{adj}A| = |A|^2$)
- Q3. (a) Both true, R explains A
- Q4. (a) $\frac{\pi}{3}$
- Q5. (b) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$
- Q6. (b) Both true, but R is not the correct explanation [$f'(0) = 0$]
- Q7. (a) 0 (odd function)
- Q8. (c) at least one corner point
- Q9. (a) Both true, R explains A
- Q10. (b) $I[(I + A)^3 = I + 7A]$
- Q11. (d) 90° (dot product = 0)
- Q12. (a) Both true, R explains A
- Q13. (c) 135 ($3^3 \times 5$)
- Q14. (a) $(0, 2)$
- Q15. (a) Both true, R explains A
- Q16. (b) $[0, 1]$
- Q17. (a) 0 (odd-order skew-symmetric)
- Q18. (d) A false, R true ($|x|$ not differentiable at 0)
- Q19. (a) $\frac{\pi}{4}$
- Q20. (a) Both true, R explains A

SECTION B

Answers

- Q21. $\frac{dy}{dx} = \frac{a \sin \theta}{1 - \cos \theta} = \cot \frac{\theta}{2}$
- Q22. $f(x) = \sqrt{x}, x = 25, \Delta x = 0.3$. $\Delta y \approx \frac{1}{2\sqrt{25}}(0.3) = 0.03$. So $\sqrt{25.3} \approx 5.03$.
- Q23. For continuity $k = \lim_{x \rightarrow 0} \frac{\sin 3x}{x} = 3$.
- Q24. $\vec{a} \times \vec{b} = 4\hat{i} - \hat{j} - 3\hat{k}$, $|\vec{a} \times \vec{b}| = \sqrt{26}$. Unit vector $= \frac{1}{\sqrt{26}}(4\hat{i} - \hat{j} - 3\hat{k})$.
- Q25. Projection $= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{2 - 2 + 23}{\sqrt{23}} = \frac{23}{\sqrt{23}}$.

SECTION C

Answers

- Q26. f is continuous on $[1, 3]$, differentiable on $(1, 3)$, and $f(1) = f(3) = 0$. $f'(c) = 2c - 4 = 0 \Rightarrow c = 2 \in (1, 3)$. Rolle's theorem verified.
- Q27. $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$ and expansion gives $(a - b)(b - c)(c - a)$. Proved.
- Q28. $\frac{d}{dx}(x - 1)(x - 2) = \frac{d}{dx}(-x^2 + 3x - 2) = -2x + 3$. $\int = -\ln|x - 1| + 2\ln|x - 2| + C$.

Q29. $\vec{cd}_1 \times \vec{cd}_2 = 3\hat{i} - \hat{j} - 7\hat{k}$, $|\vec{cd}| = \sqrt{59}$; $\vec{ca}_2 - \vec{ca}_1 = (1, 0, -1)$. SD
 $= \frac{1}{\sqrt{59}} |(1, 0, -1) \cdot (3, -1, -7)| \sqrt{59} = \frac{10}{\sqrt{59}}$ units.

Q30. $|A| = 1$; by row reduction **Misplaced &**

Q31. $\int \frac{dx}{x(x-3)^2 + 2^2} = \frac{1}{2} \tan^{-1} \frac{x}{2} - \frac{3}{2} \ln|x-3| + C$.

SECTION D

Answers

Q32. Maximise $Z = 60x + 40y$ s.t. $3x + y \leq 12$, $x + 2y \leq 10$, $x, y \geq 0$. Corner points:
 $(0, 0), (4, 0), (0, 5), \left(\frac{14}{5}, \frac{18}{5}\right)$. Z values: 0, 240, 200, 312. Maximum profit = **Rs. 312** at
 $x = 2.8, y = 3.6$.

Q33. $AX = B$ with **Misplaced &**; $|A| \neq 0$. $X = A^{-1}B$ gives $x = 1, y = 2, z = 3$.

Q34. With $r^2 = 2Rh - h^2$, $V = \frac{1}{3}\pi(2Rh - h^2)h$. $\frac{dV}{dh} = 0 \Rightarrow h = \frac{4R}{3}$ ($V'' < 0$). Maximum
 volume = $\frac{32}{27}\pi R^3$ left (= $\frac{8}{27}$ of the sphere's volume).

Q35. $\vec{cn} = \vec{cAB} \times \vec{cAC} = \hat{i} - 3\hat{j} - 6\hat{k}$. Plane: $x - 3y - 6z + 8 = 0$. Distance of
 $P(1, 2, 3) = \frac{|1 - 6 - 18 + 8|}{\sqrt{46}} = \frac{15}{\sqrt{46}}$. Angle with $2x - y + 2z + 5 = 0$:
 $\cos\theta = \frac{7}{\sqrt{46}}$, so $\theta = \cos^{-1} \frac{7}{\sqrt{46}}$.

SECTION E

Answers

Q36. (i) $V(x) = x(18 - 2x)^2$, with (ii) $\frac{dV}{dx} = 12(9 - x)(3 - x) = 0 \Rightarrow x = 3$ (rejecting $x = 9$).
 (iii) $V''(3) < 0$, so maximum. $V(3) = 3 \times 12^2 = 432$ cm³.

Q37. (i) QP is 2×1 ; defined since columns of Q (3) = rows of P (3).
 (ii) Day 1 = $50(10) + 30(15) + 20(20) = 1350$; Day 2 = $40(10) + 35(15) + 25(20) = 1425$ (Rs.).
 (iii) New Day 1 = $500 + 450 + 20(25) = 1450$ (Rs.).

Q38. (i) $v(t) = 3t^2 - 12t + 9$; $v(2) = -3$ m/s.
 (ii) $v = 0 \Rightarrow 3(t - 1)(t - 3) = 0 \Rightarrow t = 1, 3$ s.
 (iii) $a(t) = 6t - 12$, $a(1) = -6$ m/s². Moving forward where $v > 0$: $t \in (0, 1) \cup (3, \infty)$.