

**MID-TERM EXAMINATION — Class 11 Mathematics (Set G)**

Chapters: Sets, Relations & Functions, Trigonometric Functions, Complex Numbers & Quadratic Equations, Linear Inequalities, Permutations & Combinations, Binomial Theorem, Sequences & Series, Straight Lines, Conic Sections

**Max
Marks:
80**

Name: _____

Time: 3 Hours

General Instructions: All questions are compulsory. The paper has five sections — A (20×1), B (5×2), C (6×3), D (4×5) and E (3×4, case-studies). Draw neat figures where required. Use of a calculator is not permitted.

SECTION A

Multiple Choice / Assertion-Reason — 1 mark each

- Q1.** The set $(A \cap B')' \cup (B \cap C)$ is equal to: [1]
(a) $A' \cup B \cup C$
(b) $A' \cup B$
(c) $A' \cup C'$
(d) $A' \cap B$
- Q2.** If set A = multiples of 4 and set B = multiples of 6, then $A \cap B$ is the set of: [1]
(a) multiples of 2
(b) multiples of 4
(c) multiples of 12
(d) multiples of 24
- Q3.** If $n(A) = 2$, the number of relations from A to A is: [1]
(a) 4
(b) 8
(c) 16
(d) 2
- Q4.** The range of the function $f(x) = \frac{x}{|x|}$, $x \neq 0$ is: [1]
(a) $\mathbb{R}$
(b) $\{0, 1\}$
(c) $\{-1, 1\}$
(d) $[-1, 1]$
- Q5.** The value of $\cos^2 \frac{\pi}{6} - \sin^2 \frac{\pi}{6}$ is: [1]
(a) $\frac{1}{2}$
(b) $\frac{\sqrt{3}}{2}$
(c) 1
(d) $\frac{\sqrt{3}}{4}$

- Q6.** If $\sin \theta = \frac{12}{13}$ and θ is acute, then $\cos \theta$ equals: [1]
- (a) $\frac{5}{13}$
 (b) $\frac{13}{5}$
 (c) $\frac{12}{5}$
 (d) $\frac{5}{12}$
- Q7.** For a complex number z , the product $(z + 3)(\bar{z} + 3)$ equals: [1]
- (a) $|z + 3|^2$
 (b) $|z - 3|$
 (c) $z^2 + 3$
 (d) none of these
- Q8.** The value of i^{-1} is: [1]
- (a) i
 (b) $-i$
 (c) 1
 (d) -1
- Q9.** The solution set of $\frac{2x}{5} + 1 > x - 3$ is: [1]
- (a) $x < \frac{20}{3}$
 (b) $x > \frac{20}{3}$
 (c) $x < \frac{3}{20}$
 (d) $x > -\frac{20}{3}$
- Q10.** The value of ${}^8P_3$ is: [1]
- (a) 56
 (b) 336
 (c) 512
 (d) 168
- Q11.** If ${}^nC_2 = 45$, then n equals: [1]
- (a) 9
 (b) 10
 (c) 12
 (d) 15
- Q12.** The sum of the coefficients in the expansion of $(2x + 3y)^4$ is: [1]
- (a) 120
 (b) 625
 (c) 256
 (d) 81

- Q13.** The 10th term of the G.P. $1, -2, 4, -8, \dots$ is: [1]
 (a) 512
 (b) **-512**
 (c) 256
 (d) **-256**
- Q14.** If the 4th and 9th terms of a G.P. are 54 and 13122, its 6th term is: [1]
 (a) 243
 (b) 1458
 (c) 486
 (d) 729
- Q15.** The sum $1 + 3 + 5 + \dots + (2n - 1)$ for $n = 8$ is: [1]
 (a) 49
 (b) 64
 (c) 81
 (d) 72
- Q16.** The slope of the line $3x - 4y + 5 = 0$ is: [1]
 (a) $\frac{3}{4}$
 (b) **$-\frac{3}{4}$**
 (c) $\frac{4}{3}$
 (d) **3**
- Q17.** The value of y so that the line through $(3, y)$ and $(2, 7)$ is parallel to the line through $(-1, 4)$ and $(0, 6)$ is: [1]
 (a) 7
 (b) 8
 (c) 9
 (d) 10
- Q18.** The length of the latus rectum of the parabola $y^2 = 8x$ is: [1]
 (a) 2
 (b) 4
 (c) 8
 (d) 16
- Q19.** Assertion (A): The empty set is a subset of every set. Reason (R): Every set is a subset of itself. [1]
 (a) Both A and R are true and R is the correct explanation of A
 (b) Both A and R are true but R is not the correct explanation of A
 (c) A is true, R is false
 (d) A is false, R is true

- Q20.** Assertion (A): The eccentricity of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ is $\frac{4}{5}$. Reason (R): For an ellipse $c^2 = a^2 - b^2$ and $e = \frac{c}{a}$ [1]
- (a) Both A and R are true and R is the correct explanation of A
 (b) Both A and R are true but R is not the correct explanation of A
 (c) A is true, R is false
 (d) A is false, R is true

SECTION B

Very Short Answer — 2 marks each

- Q1.** If $\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of x and y . [2]
- Q2.** Find the value of $\sin \frac{31\pi}{3}$. [2]
- Q3.** Solve the inequality $|3 - 2x| \leq 5, x \in \mathbb{R}$. [2]
- Q4.** Find r if ${}^{10}P_r = 720$. [2]
- Q5.** Insert two geometric means between 3 and 24. [2]

SECTION C

Short Answer — 3 marks each

- Q1.** In a group of 65 people, 40 like tea, 10 like both tea and coffee, and 25 like neither. Find how many like coffee. [3]
- Q2.** Prove that $\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} = 2 \operatorname{cosec} x$. [3]
- Q3.** Find the conjugate of the complex number $\frac{1 + 2i}{1 - 3i}$. [3]
- Q4.** Solve the system of inequalities $3x - 7 > 2(x - 6)$ and $6 - x > 11 - 2x$, and write the solution set. [3]
- Q5.** How many 4-letter code words can be formed from the letters of the word LOGARITHM if no letter is repeated? [3]
- Q6.** Find the coefficient of x^4 in the expansion of $(1 + x)^{10}$. [3]

SECTION D

Long Answer — 5 marks each

- Q1.** Prove that $\sin 20^\circ \sin 40^\circ \sin 80^\circ = \frac{\sqrt{3}}{8}$. [5]
- Q2.** Solve the quadratic equation $x^2 - 4x + 5 = 0$ and express its roots in the form $a + ib$. [5]
- Q3.** In a G.P. the 3rd term is 24 and the 6th term is 192. Find the first term, the common ratio, and the sum of the first 8 terms. [5]
- Q4.** For the hyperbola $9x^2 - 16y^2 = 144$, find the coordinates of the vertices and foci, the eccentricity, and the length of the latus rectum. [5]

SECTION E

Case-Study Based — 4 marks each

- Q1.** In a class of 60 students, 30 offered Physics, 32 offered Chemistry and 24 offered both. (i) How many offered Physics only? (ii) How many offered Chemistry only? (iii) How many offered at least one subject? (iv) How many offered neither? [4]
- Q2.** The end posts of a straight fence are at $A(2, -3)$ and $B(-3, 5)$, forming a diameter of a circular flower bed. (i) Write the coordinates of the centre of the bed. (ii) Find the radius (leave the answer in surd form). (iii) Write the equation of the circle. [4]

- Q3.** A stadium has 30 seats in the first row, 32 in the second, 34 in the third, and so on, the seats per row forming an A.P. (i) [4]
Write the common difference. (ii) How many seats are in the 20th row? (iii) Find the total number of seats if there are 25 rows.

ANSWER KEY & MARKING SCHEME

SECTION A

Answers

- Q1. (a) $A' \cup B \cup C$
- Q2. (c) multiples of 12
- Q3. (c) 16
- Q4. (c) $\{-1, 1\}$
- Q5. (a) $\frac{1}{2}$
- Q6. (a) $\frac{5}{13}$
- Q7. (a) $|z + 3|^2$
- Q8. (b) $-i$
- Q9. (a) $x < \frac{20}{3}$
- Q10. (b) 336
- Q11. (b) 10
- Q12. (b) 625
- Q13. (b) -512
- Q14. (c) 486
- Q15. (b) 64
- Q16. (a) $\frac{3}{4}$
- Q17. (c) 9
- Q18. (c) 8
- Q19. (b) Both A and R are true but R is not the correct explanation of A
- Q20. (a) Both A and R are true and R is the correct explanation of A

SECTION B

Answers

- Q1. $\frac{x}{3} + 1 = \frac{5}{3} \Rightarrow \frac{x}{3} = \frac{2}{3} \Rightarrow x = 2; y - \frac{2}{3} = \frac{1}{3} \Rightarrow y = 1$. So $x = 2, y = 1$.
- Q2. $\frac{31\pi}{3} = 10\pi + \frac{\pi}{3}$, so $\sin \frac{31\pi}{3} = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$.
- Q3. $|3 - 2x| \leq 5 \Rightarrow -5 \leq 3 - 2x \leq 5 \Rightarrow -8 \leq -2x \leq 2 \Rightarrow -1 \leq x \leq 4$. Solution $[-1, 4]$.
- Q4. ${}^{10}P_r = \frac{10!}{(10-r)!} = 720 = 10 \times 9 \times 8 \Rightarrow r = 3$.
- Q5. Let the means be $3r, 3r^2, 3, 3r, 3r^2, 24$ is a G.P., so $3r^3 = 24 \Rightarrow r^3 = 8 \Rightarrow r = 2$. Means: 6 and 12.

SECTION C

Answers

- Q1. Those liking at least one = $65 - 25 = 40$. $n(T \cup C) = n(T) + n(C) - n(T \cap C) \Rightarrow 40 = 40 + n(C) - 10 \Rightarrow n(C) = 10$. So 10 like coffee.
- Q2. LHS

$$= \frac{\sin^2 x + (1 + \cos x)^2}{\sin x(1 + \cos x)} = \frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x(1 + \cos x)} = \frac{2 + 2\cos x}{\sin x(1 + \cos x)} = \frac{2(1 + \cos x)}{\sin x(1 + \cos x)} = \frac{2}{\sin x} = 2 \operatorname{cosec} x.$$

- Q3. $\frac{1+2i}{1-3i} = \frac{(1+2i)(1+3i)}{(1-3i)(1+3i)} = \frac{1+3i+2i+6i^2}{10} = \frac{-5+5i}{10} = -\frac{1}{2} + \frac{1}{2}i$. Conjugate = $-\frac{1}{2} - \frac{1}{2}i$.
- Q4. $3x - 7 > 2x - 12 \Rightarrow x > -5$; $6 - x > 11 - 2x \Rightarrow x > 5$. Both hold when $x > 5$. Solution $(5, \infty)$.
- Q5. LOGARITHM has 9 distinct letters; number of 4-letter code words = ${}^9P_4 = 9 \times 8 \times 7 \times 6 = 3024$.
- Q6. Coefficient of x^4 in $(1+x)^{10}$ is ${}^{10}C_4 = 210$.

SECTION D

Answers

- Q1. $\sin 20^\circ \sin 40^\circ \sin 80^\circ$: using $\sin(60-\theta) \sin \theta \sin(60+\theta) = \frac{1}{4} \sin 3\theta$ with $\theta = 20^\circ$ gives $\frac{1}{4} \sin 60^\circ = \frac{1}{4} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$.
- Q2. $x = \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm \sqrt{-4}}{2} = 2 \pm i$. Roots: $2+i$ and $2-i$.
- Q3. $ar^2 = 24$ and $ar^5 = 192 \Rightarrow r^3 = 8 \Rightarrow r = 2$; then $a \cdot 4 = 24 \Rightarrow a = 6$. $S_8 = \frac{6(2^8-1)}{2-1} = 6 \times 255 = 1530$.
- Q4. $9x^2 - 16y^2 = 144 \Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$, so $a = 4$, $b = 3$, $c = \sqrt{16+9} = 5$. Vertices $(\pm 4, 0)$; foci $(\pm 5, 0)$; eccentricity $e = \frac{5}{4}$; latus rectum = $\frac{2b^2}{a} = \frac{9}{2}$.

SECTION E

Answers

- Q1. (i) Physics only = $30 - 24 = 6$. (ii) Chemistry only = $32 - 24 = 8$. (iii) At least one = $30 + 32 - 24 = 38$. (iv) Neither = $60 - 38 = 22$.
- Q2. (i) Centre = midpoint of $AB = \left(\frac{2-3}{2}, \frac{-3+5}{2}\right) = \left(-\frac{1}{2}, 1\right)$. (ii) Radius = $\frac{1}{2}|AB| = \frac{1}{2}\sqrt{(2+3)^2 + (-3-5)^2} = \frac{1}{2}\sqrt{89}$. (iii) $(x-2)(x+3) + (y+3)(y-5) = 0$, i.e. $x^2 + y^2 + x - 2y - 21 = 0$.
- Q3. (i) Common difference $d = 2$. (ii) 20th row = $30 + 19(2) = 68$ seats. (iii) $S_{25} = \frac{25}{2}[2(30) + 24(2)] = \frac{25}{2}(108) = 1350$ seats.