

**MID-TERM EXAMINATION — Class 11 Mathematics (Set F)**

Chapters: Sets, Relations & Functions, Trigonometric Functions, Complex Numbers & Quadratic Equations, Linear Inequalities, Permutations & Combinations, Binomial Theorem, Sequences & Series, Straight Lines, Conic Sections

Max**Marks:****80**

Name: _____

Time: 3 Hours

General Instructions: All questions are compulsory. The paper has five sections — A (20×1), B (5×2), C (6×3), D (4×5) and E (3×4, case-studies). Draw neat figures where required. Use of a calculator is not permitted.

SECTION A*Multiple Choice / Assertion-Reason — 1 mark each*

- Q1.** If $A = \{2, 5\}$, the number of elements in the power set $P(A)$ is: [1]
(a) 2
(b) 4
(c) 8
(d) 16
- Q2.** In the set $A = \{1, 2, \dots, 10\}$, the relation $R = \{(a, b) \in A \times A : a = 2b\}$ has range: [1]
(a) $\{2, 4, 6, 8, 10\}$
(b) $\{1, 2, 3, 4, 5\}$
(c) $\{1, 2, \dots, 10\}$
(d) $\{2, 4, 6, 8\}$
- Q3.** The range of the function $f(x) = \frac{x-4}{|x-4|}$ is: [1]
(a) $\{4\}$
(b) $\{-4\}$
(c) $\{-1, 1\}$
(d) $\mathbb{R}$
- Q4.** If $[x]^2 - 5[x] + 6 = 0$, where $[]$ denotes the greatest integer function, then: [1]
(a) $x \in [3, 4)$
(b) $x \in [2, 3)$
(c) $x \in [2, 4)$
(d) $x \in (2, 3]$
- Q5.** The value of $\sin(45^\circ + \theta) - \cos(45^\circ - \theta)$ is: [1]
(a) $2 \cos \theta$
(b) $2 \sin \theta$
(c) 1
(d) 0
- Q6.** If $\tan \theta = \frac{1}{2}$ and $\tan \varphi = \frac{1}{3}$, then $\theta + \varphi$ equals: [1]
(a) $\frac{\pi}{6}$
(b) $\frac{\pi}{4}$
(c) 0
(d) $\frac{\pi}{3}$

- Q7.** The roots of the quadratic equation $x^2 - 5ix - 6 = 0$ are: [1]
(a) **3, 2**
(b) **$-3i, -2i$**
(c) **$3i, 2i$**
(d) none of these
- Q8.** The magnitude (modulus) of the complex number $12i - 5$ is: [1]
(a) 7
(b) 13
(c) 17
(d) 12
- Q9.** The solution set of $5 - 2x \geq -3$ is: [1]
(a) **$x \leq 4$**
(b) **$x \geq 4$**
(c) **$x \leq -4$**
(d) **$x \geq -4$**
- Q10.** The number of triangles formed by choosing vertices from 12 points, of which 7 are collinear, is: [1]
(a) 105
(b) 185
(c) 175
(d) 220
- Q11.** The number of words formed by taking 2 vowels and 3 consonants from 4 vowels and 5 consonants (arranging all five) is: [1]
(a) 60
(b) 120
(c) 7200
(d) 720
- Q12.** The middle term in the expansion of $\left(x + \frac{1}{x}\right)^{10}$ is: [1]
(a) 210
(b) 252
(c) 120
(d) 45
- Q13.** The 8th term from the end of the G.P. **3, 6, 12, ..., 12288** is: [1]
(a) 96
(b) 192
(c) 48
(d) 288
- Q14.** In a G.P., **$a = 3, a_n = 96$ and $S_n = 189$** . The value of **n** is: [1]
(a) 7
(b) 8
(c) 6
(d) 5
- Q15.** The sum of an infinite G.P. is 15 and the sum of the squares of its terms is 45. The first term is: [1]
(a) 6
(b) 7
(c) 5
(d) 9

- Q16.** The equation of the line through $(1, 2)$ perpendicular to $x + y + 1 = 0$ is: [1]
 (a) $y - x + 1 = 0$
 (b) $y - x - 1 = 0$
 (c) $y - x + 2 = 0$
 (d) $y - x - 2 = 0$
- Q17.** The new coordinates of the point $(3, -5)$ when the origin is shifted to $(-3, -2)$ are: [1]
 (a) $(6, 3)$
 (b) $(6, -3)$
 (c) $(-6, 3)$
 (d) $(-6, -3)$
- Q18.** The focal distance of a point P on the parabola $y^2 = 12x$ is 4. The abscissa of P is: [1]
 (a) -1
 (b) 1
 (c) -2
 (d) 2
- Q19.** Assertion (A): The number of relations from a set A to a set B with $n(A) = m$, $n(B) = k$ is 2^{mk} . Reason (R): A relation from A to B is a subset of $A \times B$. [1]
 (a) Both A and R are true and R is the correct explanation of A
 (b) Both A and R are true but R is not the correct explanation of A
 (c) A is true, R is false
 (d) A is false, R is true
- Q20.** Assertion (A): The hyperbola with vertices $(\pm 2, 0)$ and foci $(\pm 3, 0)$ is $\frac{x^2}{4} - \frac{y^2}{5} = 1$. Reason (R): For a hyperbola $b^2 = c^2 - a^2$. [1]
 (a) Both A and R are true and R is the correct explanation of A
 (b) Both A and R are true but R is not the correct explanation of A
 (c) A is true, R is false
 (d) A is false, R is true

SECTION B

Very Short Answer — 2 marks each

- Q1.** Find the domain and range of the function $f(x) = \sqrt{x-1}$. [2]
- Q2.** Find the value of $\cos 75^\circ$. [2]
- Q3.** Solve the inequality $5(2-x) \geq 2(4-x)$, $x \in \mathbb{R}$. [2]
- Q4.** Find n if ${}^nP_4 = 20 {}^nP_2$. [2]
- Q5.** How many terms of the G.P. $2, 6, 18, \dots$ are needed to make the sum 728? [2]

SECTION C

Short Answer — 3 marks each

- Q1.** In a survey of 25 students, 15 like Mathematics, 12 like Science and 5 like both. Find the number who like only Mathematics, only Science, and at least one of the two subjects. [3]
- Q2.** Prove that $\frac{1 - \cos 2x + \sin x}{\sin 2x + \cos x} = \tan x$. [3]
- Q3.** Express $\frac{3-2i}{2+i}$ in the form $a+ib$. [3]
- Q4.** Solve the inequality $-3 \leq \frac{4-3x}{2} \leq 6$, $x \in \mathbb{R}$, and represent the solution on the number line. [3]
- Q5.** In how many distinct ways can the letters of the word MISSISSIPPI be arranged? [3]
- Q6.** Find the term independent of x in the expansion of $\left(2x^2 - \frac{1}{x}\right)^6$. [3]

SECTION D*Long Answer — 5 marks each*

- Q1.** Prove that $\cos 20^\circ \cos 40^\circ \cos 80^\circ = \frac{1}{8}$. [5]
- Q2.** Solve the quadratic equation $2x^2 - 4x + 3 = 0$ and express its roots in the form $a + ib$. [5]
- Q3.** The sum of the first three terms of a G.P. is 13 and their product is 27. Find the three terms. [5]
- Q4.** Find the equation of the hyperbola with foci $(0, \pm 12)$ and length of latus rectum 36. [5]

SECTION E*Case-Study Based — 4 marks each*

- Q1.** In a colony of 400 families, 250 read newspaper A, 200 read newspaper B and 100 read both. (i) How many read A only? (ii) How many read B only? (iii) How many read at least one newspaper? (iv) How many read neither? [4]
- Q2.** A person saves Rs. 500 in the first month and increases the saving by Rs. 100 every month, so the monthly savings **500, 600, 700, ...** [4]
form an A.P. (i) Write the common difference. (ii) How much is saved in the 12th month? (iii) Find the total amount saved in one year (12 months).
- Q3.** The reflector of a torch is a parabola $y^2 = 8x$ (vertex at the origin, opening rightwards). (i) Write the coordinates of the focus. (ii) Write the length of the latus rectum. (iii) Write the equation of the directrix. [4]

ANSWER KEY & MARKING SCHEME

SECTION A

Answers

- Q1. (b) 4
- Q2. (b) $\{1, 2, 3, 4, 5\}$
- Q3. (c) $\{-1, 1\}$
- Q4. (c) $x \in [2, 4)$
- Q5. (d) 0
- Q6. (b) $\frac{\pi}{4}$
- Q7. (c) $3i, 2i$
- Q8. (b) 13
- Q9. (a) $x \leq 4$
- Q10. (b) 185
- Q11. (c) 7200
- Q12. (b) 252
- Q13. (a) 96
- Q14. (c) 6
- Q15. (c) 5
- Q16. (b) $y - x - 1 = 0$
- Q17. (b) $(6, -3)$
- Q18. (b) 1
- Q19. (a) Both A and R are true and R is the correct explanation of A
- Q20. (a) Both A and R are true and R is the correct explanation of A

SECTION B

Answers

- Q1. For $\sqrt{x-1}$ to be real, $x-1 \geq 0 \Rightarrow x \geq 1$. Domain = $[1, \infty)$; range = $[0, \infty)$.
- Q2. $\cos 75^\circ = \cos(45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$.
- Q3. $10 - 5x \geq 8 - 2x \Rightarrow 2 \geq 3x \Rightarrow x \leq \frac{2}{3}$. Solution $\left(-\infty, \frac{2}{3}\right]$.
- Q4. ${}^nP_4 = 20 {}^nP_2 \Rightarrow n(n-1)(n-2)(n-3) = 20n(n-1) \Rightarrow (n-2)(n-3) = 20 \Rightarrow n^2 - 5n - 14 = 0 \Rightarrow (n-7)(n+2) = 0 \Rightarrow n = 7$.
- Q5. $S_n = \frac{2(3^n - 1)}{3 - 1} = 3^n - 1 = 728 \Rightarrow 3^n = 729 = 3^6 \Rightarrow n = 6$ terms.

SECTION C

Answers

- Q1. Only Maths = $15 - 5 = 10$; only Science = $12 - 5 = 7$; at least one = $15 + 12 - 5 = 22$.
- Q2. Numerator = $1 - \cos 2x + \sin x = 2 \sin^2 x + \sin x = \sin x(2 \sin x + 1)$; denominator = $\sin 2x + \cos x = 2 \sin x \cos x + \cos x = \cos x(2 \sin x + 1)$. Ratio = $\frac{\sin x}{\cos x} = \tan x$.
- Q3. $\frac{3-2i}{2+i} = \frac{(3-2i)(2-i)}{(2+i)(2-i)} = \frac{6-3i-4i+2i^2}{5} = \frac{4-7i}{5} = \frac{4}{5} - \frac{7}{5}i$.
- Q4. $-3 \leq \frac{4-3x}{2} \Rightarrow -6 \leq 4-3x \Rightarrow x \leq \frac{10}{3}$; and $\frac{4-3x}{2} \leq 6 \Rightarrow 4-3x \leq 12 \Rightarrow x \geq -\frac{8}{3}$. Solution $\left[-\frac{8}{3}, \frac{10}{3}\right]$.
- Q5. MISSISSIPPI has 11 letters: I $\times 4$, S $\times 4$, P $\times 2$, M $\times 1$. Number of arrangements = $\frac{11!}{4!4!2!} = 34650$.
- Q6. $T_{r+1} = {}^6C_r (2x^2)^{6-r} \left(-\frac{1}{x}\right)^r = {}^6C_r 2^{6-r} (-1)^r x^{12-3r}$. Independent when $12-3r = 0 \Rightarrow r = 4$: ${}^6C_4 \cdot 2^2 = 15 \times 4 = 60$.

SECTION D

Answers

- Q1. $\cos 20^\circ \cos 40^\circ \cos 80^\circ$: multiply and divide by $2 \sin 20^\circ$. Using $2 \sin \theta \cos \theta = \sin 2\theta$ repeatedly gives $\frac{\sin 160^\circ}{8 \sin 20^\circ} = \frac{\sin 20^\circ}{8 \sin 20^\circ} = \frac{1}{8}$.
- Q2. $x = \frac{4 \pm \sqrt{16-24}}{4} = \frac{4 \pm \sqrt{-8}}{4} = 1 \pm \frac{\sqrt{2}}{2}i$.
- Q3. Let the terms be $\frac{a}{r}, a, ar$. Product = $a^3 = 27 \Rightarrow a = 3$. Sum = $3 \left(\frac{1}{r} + 1 + r \right) = 13 \Rightarrow \frac{1}{r} + r = \frac{10}{3} \Rightarrow 3r^2 - 10r + 3 = 0 \Rightarrow r = 3$ or $\frac{1}{3}$. Terms: 1, 3, 9.
- Q4. Foci on the y -axis: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ with $c = 12$. LR = $\frac{2b^2}{a} = 36 \Rightarrow b^2 = 18a$; and $a^2 + b^2 = 144 \Rightarrow a^2 + 18a - 144 = 0 \Rightarrow a = 6$, so $a^2 = 36, b^2 = 108$. Equation: $\frac{y^2}{36} - \frac{x^2}{108} = 1$.

SECTION E

Answers

- Q1. (i) A only = $250 - 100 = 150$. (ii) B only = $200 - 100 = 100$. (iii) At least one = $250 + 200 - 100 = 350$. (iv) Neither = $400 - 350 = 50$.
- Q2. (i) Common difference $d = 100$. (ii) 12th month = $500 + 11(100) = 1600$. (iii) $S_{12} = \frac{12}{2} [2(500) + 11(100)] = 6(2100) = 12600$.
- Q3. Compare $y^2 = 8x$ with $y^2 = 4ax \Rightarrow a = 2$. (i) Focus $(2, 0)$. (ii) Latus rectum = 8. (iii) Directrix $x = -2$.