



MID-TERM EXAMINATION — Class 11 Mathematics (Set E)

Chapters: Sets, Relations & Functions, Trigonometric Functions, Complex Numbers & Quadratic Equations, Linear Inequalities, Permutations & Combinations, Binomial Theorem, Sequences & Series, Straight Lines, Conic Sections

**Max
Marks:**
80

Name: _____

Time: 3 Hours

General Instructions: All questions are compulsory. The paper has five sections — A (20×1), B (5×2), C (6×3), D (4×5) and E (3×4, case-studies). Draw neat figures where required. Use of a calculator is not permitted.

SECTION A

Multiple Choice / Assertion-Reason — 1 mark each

- Q1.** The number of subsets of a set containing 5 elements is: [1]
(a) 10
(b) 25
(c) 32
(d) 31
- Q2.** If $n(A) = 3$ and $n(B) = 2$, then the number of non-empty relations from A to B is: [1]
(a) 64
(b) 63
(c) 8
(d) 6
- Q3.** For two disjoint sets A and B with $n(A) = 3$ and $n(B) = 5$, $n(A \cap B)$ equals: [1]
(a) 0
(b) 3
(c) 5
(d) 8
- Q4.** The domain of the function $f(x) = \sqrt{a^2 - x^2}$ ($a > 0$) is: [1]
(a) $(-a, a)$
(b) $[-a, a]$
(c) $[0, a]$
(d) $(-a, 0]$

- Q5.** The angle described by the minute hand of a clock in 20 minutes is: [1]
- (a) $\frac{\pi}{6}$
 - (b) $\frac{\pi}{3}$
 - (c) $\frac{2\pi}{3}$
 - (d) $\frac{\pi}{4}$
- Q6.** The minimum value of $3 \cos x + 4 \sin x + 8$ is: [1]
- (a) 5
 - (b) 3
 - (c) 7
 - (d) 13
- Q7.** The conjugate of the complex number $i^3 - 4$ is: [1]
- (a) $-4 - i$
 - (b) $4 - i$
 - (c) $-4 + i$
 - (d) $4 + i$
- Q8.** The modulus of $\frac{1-i}{1+i}$ is: [1]
- (a) 1
 - (b) 2
 - (c) $\sqrt{2}$
 - (d) $\frac{1}{2}$
- Q9.** The solution set of $3x - 2 < 2x + 1$ is: [1]
- (a) $x < 3$
 - (b) $x > 3$
 - (c) $x < 1$
 - (d) $x > 1$
- Q10.** Ten students take part in a competition with three distinct prizes. The number of ways the prizes can be awarded is: [1]
- (a) 720
 - (b) 120
 - (c) 30
 - (d) 1000

- Q11.** If ${}^nC_{12} = {}^nC_8$, then n equals: [1]
- (a) 20
 - (b) 12
 - (c) 6
 - (d) 4
- Q12.** The number of terms in the expansion of $(x + a)^{51} - (x - a)^{51}$ after simplification is: [1]
- (a) 102
 - (b) 25
 - (c) 26
 - (d) 51
- Q13.** If the n th term of a G.P. is 2^n , the sum of its first 6 terms is: [1]
- (a) 124
 - (b) 126
 - (c) 190
 - (d) 254
- Q14.** Which term of the G.P. 5, 10, 20, 40, ... is 5120? [1]
- (a) 9th
 - (b) 10th
 - (c) 11th
 - (d) 12th
- Q15.** The sum of an infinite G.P. is 8 and its second term is 2. The common ratio is: [1]
- (a) $\frac{1}{2}$
 - (b) $\frac{1}{4}$
 - (c) $\frac{2}{3}$
 - (d) $\frac{3}{4}$
- Q16.** The equation of the line whose intercepts on the x - and y -axes are -2 and 3 respectively is: [1]
- (a) $3x - 2y = 6$
 - (b) $3x - 2y + 6 = 0$
 - (c) $3x + 2y = 6$
 - (d) $3x + 2y = -6$

Q17. The distance of the point $(3, -5)$ from the line $3x - 4y - 26 = 0$ is: [1]

- (a) $\frac{3}{7}$
- (b) $\frac{2}{5}$
- (c) $\frac{7}{5}$
- (d) $\frac{3}{5}$

Q18. The centre of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is: [1]

- (a) $(-3, 2)$
- (b) $(3, 2)$
- (c) $(3, -2)$
- (d) $(-3, -2)$

Q19. Assertion (A): For any set A , $A \cup A' = U$. Reason (R): A and A' have no element in common. [1]

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is not the correct explanation of A
- (c) A is true, R is false
- (d) A is false, R is true

Q20. Assertion (A): For the parabola $y^2 = -12x$, the focus is $(-3, 0)$. Reason (R): For $y^2 = -4ax$ the directrix is $x = a$. [1]

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is not the correct explanation of A
- (c) A is true, R is false
- (d) A is false, R is true

SECTION B

Very Short Answer — 2 marks each

Q1. If $A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$, find the sets A and B . [2]

Q2. Find the value of $\tan 75^\circ - \cot 75^\circ$. [2]

Q3. Solve the inequality $3(1 - x) < 2(x + 4)$, $x \in \mathbb{R}$. [2]

Q4. Find n if ${}^nC_{10} = {}^nC_{12}$. [2]

Q5. Which term of the G.P. $2, 6, 18, 54, \dots$ is 486? [2]

SECTION C

Short Answer — 3 marks each

Q1. Write the set $A = \{x : x \in \mathbb{Z}, x^2 < 20\}$ in roster form. [3]

Q2. If $\sin \theta + \operatorname{cosec} \theta = 2$, find the value of $\sin^2 \theta + \operatorname{cosec}^2 \theta$. [3]

Q3. Find the modulus and the argument of the complex number $z = -1 - i\sqrt{3}$. [3]

- Q4.** Solve the system of inequalities $2(x - 1) < x + 5$ and $3(x + 2) > 2 - x$, and write the solution set. [3]
- Q5.** How many words, with or without meaning, can be formed using all the letters of the word EQUATION, using each letter exactly once? [3]
- Q6.** Find the coefficient of x^6 in the expansion of $(2 + x)^9$. [3]

SECTION D

Long Answer — 5 marks each

- Q1.** Prove that $\cos^2 x + \cos^2\left(x + \frac{2\pi}{3}\right) + \cos^2\left(x - \frac{2\pi}{3}\right) = \frac{3}{2}$. [5]
- Q2.** Solve the quadratic equation $x^2 - 2x + 2 = 0$ and express its roots in the form $a + ib$. [5]
- Q3.** The sum of the first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the three terms. [5]
- Q4.** Find the coordinates of the foci and the vertices, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. [5]

SECTION E

Case-Study Based — 4 marks each

- Q1.** In a survey of 200 students, 120 like cricket, 90 like football and 40 like both games. (i) How many like cricket only? (ii) How many like football only? (iii) How many like at least one game? (iv) How many like neither game? [4]
- Q2.** A ball is dropped from a height of 64 m and on each bounce it rises to half the previous height, so the rebound heights **32, 16, 8, ...** form a G.P. (i) Write the common ratio. (ii) Find the height after the 4th bounce. (iii) Find the total distance travelled by the ball before coming to rest (up-and-down). [4]
- Q3.** A circular park is represented by the equation $x^2 + y^2 - 6x + 4y - 12 = 0$ on a coordinate map. (i) Write the coordinates of the centre. (ii) Write the radius. (iii) Does the point **(3, 3)** lie inside, on, or outside the park? Justify. [4]

ANSWER KEY & MARKING SCHEME

SECTION A

Answers

- Q1. (c) 32
- Q2. (b) 63
- Q3. (a) 0
- Q4. (b) $[-a, a]$
- Q5. (c) $\frac{2\pi}{3}$
- Q6. (b) 3
- Q7. (c) $-4 + i$
- Q8. (a) 1
- Q9. (a) $x < 3$
- Q10. (a) 720
- Q11. (a) 20
- Q12. (c) 26
- Q13. (b) 126
- Q14. (c) 11th
- Q15. (a) $\frac{1}{2}$
- Q16. (b) $3x - 2y + 6 = 0$
- Q17. (d) $\frac{3}{5}$
- Q18. (c) $(3, -2)$
- Q19. (b) Both A and R are true but R is not the correct explanation of A
- Q20. (c) A is true, R is false

SECTION B

Answers

- Q1. $A = \text{set of first components} = \{a, b\}; B = \text{set of second components} = \{1, 2\}.$
- Q2. $\tan 75^\circ = 2 + \sqrt{3}$ and $\cot 75^\circ = 2 - \sqrt{3}$, so $\tan 75^\circ - \cot 75^\circ = 2\sqrt{3}.$
- Q3. $3 - 3x < 2x + 8 \Rightarrow -5 < 5x \Rightarrow x > -1.$ Solution $(-1, \infty).$
- Q4. ${}^nC_{10} = {}^nC_{12} \Rightarrow 10 + 12 = n \Rightarrow n = 22.$
- Q5. $2 \cdot 3^{n-1} = 486 \Rightarrow 3^{n-1} = 243 = 3^5 \Rightarrow n - 1 = 5 \Rightarrow n = 6.$ So the 6th term.

SECTION C

Answers

- Q1. $x^2 < 20$ with $x \in \mathbb{Z}$ gives $x = 0, \pm 1, \pm 2, \pm 3, \pm 4.$ Roster: $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}.$

- Q2. $\sin \theta + \operatorname{cosec} \theta = 2 \Rightarrow \sin \theta + \frac{1}{\sin \theta} = 2 \Rightarrow \sin \theta = 1$. Then $\sin^2 \theta + \operatorname{cosec}^2 \theta = 1 + 1 = 2$.
- Q3. $|z| = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$. z lies in the third quadrant; the reference angle is $\frac{\pi}{3}$, so principal argument $= -\left(\pi - \frac{\pi}{3}\right) = -\frac{2\pi}{3}$.
- Q4. $2(x-1) < x+5 \Rightarrow x < 7$; $3(x+2) > 2-x \Rightarrow 4x > -4 \Rightarrow x > -1$. Solution $(-1, 7)$.
- Q5. EQUATION has 8 distinct letters, so the number of words $= 8! = 40320$.
- Q6. $T_{r+1} = {}^9C_r 2^{9-r} x^r$. For $x^6, r = 6$: coefficient $= {}^9C_6 \cdot 2^3 = 84 \times 8 = 672$.

SECTION D

Answers

- Q1. Using $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$, the sum $= \frac{3}{2} + \frac{1}{2} \left[\cos 2x + \cos\left(2x + \frac{4\pi}{3}\right) + \cos\left(2x - \frac{4\pi}{3}\right) \right]$. The three cosines sum to 0 (angles 120° apart), so the total $= \frac{3}{2}$.
- Q2. $x = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$. Roots: $1 + i$ and $1 - i$.
- Q3. Let the terms be $\frac{a}{r}, a, ar$. Product $= a^3 = 1 \Rightarrow a = 1$. Sum $= \frac{1}{r} + 1 + r = \frac{39}{10} \Rightarrow \frac{1}{r} + r = \frac{29}{10} \Rightarrow 10r^2 - 29r + 10 = 0 \Rightarrow r = \frac{5}{2}$ or $\frac{2}{5}$. Terms: $\frac{2}{5}, 1, \frac{5}{2}$.
- Q4. $a^2 = 25, b^2 = 9 \Rightarrow a = 5, b = 3, c = \sqrt{25-9} = 4$. Foci $(\pm 4, 0)$; vertices $(\pm 5, 0)$; eccentricity $e = \frac{4}{5}$; latus rectum $= \frac{2b^2}{a} = \frac{18}{5}$.

SECTION E

Answers

- Q1. (i) Cricket only $= 120 - 40 = 80$. (ii) Football only $= 90 - 40 = 50$. (iii) At least one $= 120 + 90 - 40 = 170$. (iv) Neither $= 200 - 170 = 30$.
- Q2. (i) Common ratio $r = \frac{1}{2}$. (ii) Height after 4th bounce $= 64\left(\frac{1}{2}\right)^4 = 4$ m. (iii) Total distance $= 64 + 2(32 + 16 + 8 + \dots) = 64 + 2 \times \frac{32}{1 - \frac{1}{2}} = 64 + 128 = 192$ m.
- Q3. (i) Centre $(3, -2)$. (ii) $r = \sqrt{3^2 + (-2)^2 + 12} = \sqrt{25} = 5$. (iii) Distance of $(3, 3)$ from centre $= \sqrt{0 + 25} = 5 = r$, so the point lies exactly on the boundary of the park.