

**MID-TERM EXAMINATION — Class 11 Mathematics (Set D)**

Chapters: Sets, Relations & Functions, Trigonometric Functions, Complex Numbers & Quadratic Equations, Linear Inequalities, Permutations & Combinations, Binomial Theorem, Sequences & Series, Straight Lines, Conic Sections

Max**Marks:****80**

Name: _____

Time: 3 Hours

General Instructions: All questions are compulsory. The paper has five sections — A (20×1), B (5×2), C (6×3), D (4×5) and E (3×4, case-studies). Draw neat figures where required. Use of a calculator is not permitted.

SECTION A*Multiple Choice / Assertion-Reason — 1 mark each*

Q1. The number of subsets of a set having 6 elements is:

[1]

- (a) 36
- (b) 64
- (c) 12
- (d) 128

Q2. If $n(A) = 3$, the number of functions from A to A is:

[1]

- (a) 9
- (b) 27
- (c) 6
- (d) 8

Q3. The domain of the function $f(x) = \sqrt{x-2}$ is:

[1]

- (a) $(2, \infty)$
- (b) $[2, \infty)$
- (c) $\mathbb{R}$
- (d) $(-\infty, 2]$

Q4. The range of the function $f(x) = \sin x$ is:

[1]

- (a) $\mathbb{R}$
- (b) $[-1, 1]$
- (c) $[0, 1]$
- (d) $(-1, 1)$

Q5. The value of $\sin 30^\circ + \cos 60^\circ$ is: [1]

- (a) $\frac{1}{2}$
- (b) 1
- (c) $\frac{\sqrt{3}}{2}$
- (d) 0

Q6. If $\cot \theta = 1$ and θ is acute, then $\cos \theta$ equals: [1]

- (a) $\frac{1}{\sqrt{2}}$
- (b) $\frac{\sqrt{3}}{2}$
- (c) $\frac{1}{2}$
- (d) 1

Q7. The value of i^{100} is: [1]

- (a) i
- (b) $-i$
- (c) 1
- (d) -1

Q8. The real part of $(2 + 3i)^2$ is: [1]

- (a) -5
- (b) 4
- (c) 13
- (d) 12

Q9. The solution set of $5 - 2x \leq 11$ is: [1]

- (a) $x \leq -3$
- (b) $x \geq -3$
- (c) $x \leq 3$
- (d) $x \geq 3$

Q10. The number of arrangements of the letters of the word APPLE is: [1]

- (a) 120
- (b) 60
- (c) 24
- (d) 20

Q11. The value of ${}^9C_7$ is: [1]

- (a) 36
- (b) 72
- (c) 84
- (d) 18

- Q12.** The number of terms in the expansion of $(2x + 3y)^{12}$ is: [1]
- (a) 11
 - (b) 12
 - (c) 13
 - (d) 14
- Q13.** The 10th term of the A.P. $1, 3, 5, \dots$ is: [1]
- (a) 17
 - (b) 19
 - (c) 21
 - (d) 20
- Q14.** The geometric mean of 2 and 8 is: [1]
- (a) 4
 - (b) 5
 - (c) 6
 - (d) 16
- Q15.** The sum of the first 6 terms of the G.P. $1, 2, 4, 8, \dots$ is: [1]
- (a) 31
 - (b) 63
 - (c) 127
 - (d) 64
- Q16.** The angle that a line of slope 1 makes with the positive x -axis is: [1]
- (a) 30°
 - (b) 45°
 - (c) 60°
 - (d) 90°
- Q17.** The midpoint of the segment joining $(2, 4)$ and $(6, 8)$ is: [1]
- (a) $(4, 6)$
 - (b) $(8, 12)$
 - (c) $(3, 4)$
 - (d) $(4, 4)$
- Q18.** The eccentricity of a parabola is: [1]
- (a) 0
 - (b) 1
 - (c) < 1
 - (d) > 1

Q19. Assertion (A): The empty set is a subset of every set. Reason (R): Every set is a subset of itself. [1]

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is not the correct explanation of A
- (c) A is true, R is false
- (d) A is false, R is true

Q20. Assertion (A): The equation $x^2 + y^2 = 25$ represents a circle of radius 5. Reason (R): The equation of a circle with centre $(0, 0)$ and radius r is $x^2 + y^2 = r^2$. [1]

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is not the correct explanation of A
- (c) A is true, R is false
- (d) A is false, R is true

SECTION B

Very Short Answer — 2 marks each

Q1. If $A = \{1, 2\}$ and $B = \{3, 4\}$, find the number of subsets of $A \times B$. [2]

Q2. Find the value of $\tan 15^\circ$. [2]

Q3. Solve the inequality $4x + 3 \geq 2x - 1$, $x \in \mathbb{R}$. [2]

Q4. Find n if ${}^nC_3 = {}^nC_2$. [2]

Q5. Find the 4th term of the G.P. whose first term is 2 and common ratio is 3. [2]

SECTION C

Short Answer — 3 marks each

Q1. In a group of 40 students, 22 play chess, 18 play carrom and 6 play both. Find how many students play at least one of the two games, and how many play neither. [3]

Q2. Prove that $\frac{1 + \cos 2x}{\sin 2x} = \cot x$. [3]

Q3. Find the modulus and the argument of the complex number $z = 1 - i$. [3]

Q4. Solve the inequality $2 \leq 3x - 4 \leq 11$, $x \in \mathbb{R}$, and represent the solution on the number line. [3]

Q5. In how many ways can 5 boys and 3 girls be seated in a row so that no two girls sit together? [3]

Q6. Find the coefficient of x^2 in the expansion of $(1 + x)^{10}$. [3]

SECTION D

Long Answer — 5 marks each

Q1. Prove that $\cos(A + B) \cos(A - B) = \cos^2 A - \sin^2 B$. [5]

Q2. Solve the quadratic equation $x^2 + x + 2 = 0$ and express its roots in the form $a + ib$. [5]

Q3. Find the sum of the first n terms of the series $2 + 5 + 10 + 17 + \dots$ (whose k th term is $k^2 + 1$), and hence find the sum of the first 5 terms. [5]

Q4. For the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$, find the coordinates of the foci, the eccentricity and the length of the latus rectum. [5]

SECTION E**Case-Study Based — 4 marks each**

- Q1.** In a survey of 120 families, 65 read newspaper P, 55 read newspaper Q and 30 read both. (i) How many read P only? (ii) How many read Q only? (iii) How many read at least one newspaper? (iv) How many read neither? **[4]**
- Q2.** A theatre has 20 seats in the first row, 22 in the second, 24 in the third, and so on, the number of seats in each row forming an A.P. (i) Write the common difference. (ii) How many seats are in the 10th row? (iii) If there are 15 rows, find the total number of seats in the theatre. **[4]**
- Q3.** A cable of a suspension bridge hangs as a parabola $x^2 = 40y$ (origin at the lowest point). (i) Write the coordinates of the focus. (ii) Write the length of the latus rectum. (iii) Write the equation of the directrix. **[4]**

ANSWER KEY & MARKING SCHEME

SECTION A

Answers

- Q1. (b) 64
- Q2. (b) 27
- Q3. (b) $[2, \infty)$
- Q4. (b) $[-1, 1]$
- Q5. (b) 1
- Q6. (a) $\frac{1}{\sqrt{2}}$
- Q7. (c) 1
- Q8. (a) -5
- Q9. (b) $x \geq -3$
- Q10. (b) 60
- Q11. (a) 36
- Q12. (c) 13
- Q13. (b) 19
- Q14. (a) 4
- Q15. (b) 63
- Q16. (b) 45°
- Q17. (a) $(4, 6)$
- Q18. (b) 1
- Q19. (b) Both A and R are true but R is not the correct explanation of A
- Q20. (a) Both A and R are true and R is the correct explanation of A

SECTION B

Answers

- Q1. $n(A \times B) = 2 \times 2 = 4$, so the number of subsets $= 2^4 = 16$.
- Q2. $\tan 15^\circ = \tan(45^\circ - 30^\circ) = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3}$.
- Q3. $4x + 3 \geq 2x - 1 \Rightarrow 2x \geq -4 \Rightarrow x \geq -2$. Solution $[-2, \infty)$.
- Q4. ${}^nC_3 = {}^nC_2 \Rightarrow \frac{n-2}{3} = 1 \Rightarrow n-2 = 3 \Rightarrow n = 5$.
- Q5. $T_4 = ar^3 = 2 \cdot 3^3 = 2 \times 27 = 54$.

SECTION C

Answers

- Q1. $n(\text{at least one}) = 22 + 18 - 6 = 34$; neither $= 40 - 34 = 6$.
- Q2. $\frac{1 + \cos 2x}{\sin 2x} = \frac{2 \cos^2 x}{2 \sin x \cos x} = \frac{\cos x}{\sin x} = \cot x$.

- Q3. $|z| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$. $z = 1 - i$ lies in the fourth quadrant, so argument $= -\frac{\pi}{4}$.
- Q4. $2 \leq 3x - 4 \Rightarrow 6 \leq 3x \Rightarrow x \geq 2$; $3x - 4 \leq 11 \Rightarrow x \leq 5$. Solution $[2, 5]$.
- Q5. Seat the 5 boys: $5! = 120$ ways, creating 6 gaps. Choose and arrange 3 girls in the gaps: ${}^6P_3 = 120$ ways. Total $= 120 \times 120 = 14400$.
- Q6. Coefficient of x^2 in $(1+x)^{10}$ is ${}^{10}C_2 = 45$.

SECTION D

Answers

- Q1. $\cos(A+B)\cos(A-B) = (\cos A \cos B - \sin A \sin B)(\cos A \cos B + \sin A \sin B) = \cos^2 A \cos^2 B - \sin^2 A \sin^2 B$. Writing $\cos^2 B = 1 - \sin^2 B$ and $\sin^2 A = 1 - \cos^2 A$:
 $= \cos^2 A(1 - \sin^2 B) - (1 - \cos^2 A)\sin^2 B = \cos^2 A - \sin^2 B$.
- Q2. $x = \frac{-1 \pm \sqrt{1-8}}{2} = \frac{-1 \pm \sqrt{-7}}{2} = -\frac{1}{2} \pm \frac{\sqrt{7}}{2}i$.
- Q3. $S_n = \sum_{k=1}^n (k^2 + 1) = \frac{n(n+1)(2n+1)}{6} + n$. For $n = 5$: $\frac{5 \cdot 6 \cdot 11}{6} + 5 = 55 + 5 = 60$.
- Q4. Compare with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$: $a = 4$, $b = 3$, $c = \sqrt{a^2 + b^2} = \sqrt{25} = 5$. Foci $(\pm 5, 0)$; eccentricity $e = \frac{c}{a} = \frac{5}{4}$; latus rectum $= \frac{2b^2}{a} = \frac{2 \cdot 9}{4} = \frac{9}{2}$.

SECTION E

Answers

- Q1. (i) P only $= 65 - 30 = 35$. (ii) Q only $= 55 - 30 = 25$. (iii) At least one $= 65 + 55 - 30 = 90$. (iv) Neither $= 120 - 90 = 30$.
- Q2. (i) Common difference $d = 2$. (ii) 10th row $= 20 + 9(2) = 38$ seats. (iii) $S_{15} = \frac{15}{2}[2(20) + 14(2)] = \frac{15}{2}(68) = 510$ seats.
- Q3. Compare $x^2 = 40y$ with $x^2 = 4ay \Rightarrow a = 10$. (i) Focus $(0, 10)$. (ii) Latus rectum $= 40$. (iii) Directrix $y = -10$.