



MID-TERM EXAMINATION — Class 11 Mathematics (Set C)

Chapters: Sets, Relations & Functions, Trigonometric Functions, Complex Numbers & Quadratic Equations, Linear Inequalities, Permutations & Combinations, Binomial Theorem, Sequences & Series, Straight Lines, Conic Sections

**Max
Marks:**
80

Name: _____

Time: 3 Hours

General Instructions: All questions are compulsory. The paper has five sections — A (20×1), B (5×2), C (6×3), D (4×5) and E (3×4, case-studies). Draw neat figures where required. Use of a calculator is not permitted.

SECTION A

Multiple Choice / Assertion-Reason — 1 mark each

- Q1.** If a set A has 4 elements, the number of subsets of A is: [1]
(a) 8
(b) 12
(c) 16
(d) 4
- Q2.** If $A = \{a, b\}$, the number of relations from A to A is: [1]
(a) 4
(b) 8
(c) 16
(d) 2
- Q3.** The domain of the function $f(x) = \frac{1}{x^2 - 1}$ is: [1]
(a) $\mathbb{R}$
(b) $\mathbb{R} - \{1\}$
(c) $\mathbb{R} - \{-1, 1\}$
(d) $\mathbb{R} - \{0\}$
- Q4.** The range of the function $f(x) = x^2$ is: [1]
(a) $\mathbb{R}$
(b) $(0, \infty)$
(c) $[0, \infty)$
(d) $(-\infty, 0]$

- Q5.** The value of $\cos(-60^\circ)$ is: [1]
- (a) $\frac{1}{2}$
 - (b) $-\frac{1}{2}$
 - (c) $\frac{\sqrt{3}}{2}$
 - (d) 1
- Q6.** The value of $\tan \frac{\pi}{4} + \cot \frac{\pi}{4}$ is: [1]
- (a) 0
 - (b) 1
 - (c) 2
 - (d) $\frac{1}{2}$
- Q7.** The value of i^{25} is: [1]
- (a) 1
 - (b) -1
 - (c) i
 - (d) $-i$
- Q8.** The modulus of the complex number $3 - 4i$ is: [1]
- (a) 5
 - (b) 7
 - (c) $\sqrt{7}$
 - (d) 25
- Q9.** The solution set of $2(x - 1) < x + 2$ is: [1]
- (a) $x < 4$
 - (b) $x > 4$
 - (c) $x < 2$
 - (d) $x > 2$
- Q10.** The number of arrangements of the letters of the word NUMBER is: [1]
- (a) 120
 - (b) 360
 - (c) 720
 - (d) 24
- Q11.** The value of ${}^7C_3$ is: [1]
- (a) 21
 - (b) 35
 - (c) 42
 - (d) 210

- Q12.** The middle term in the expansion of $(a + b)^6$ is the: [1]
- (a) 3rd term
 - (b) 4th term
 - (c) 5th term
 - (d) 6th term
- Q13.** The 7th term of the A.P. **4, 9, 14, ...** is: [1]
- (a) 29
 - (b) 34
 - (c) 39
 - (d) 44
- Q14.** The geometric mean of 3 and 27 is: [1]
- (a) 9
 - (b) 15
 - (c) 12
 - (d) 81
- Q15.** The common ratio of the G.P. **2, 6, 18, ...** is: [1]
- (a) 2
 - (b) 3
 - (c) 4
 - (d) 6
- Q16.** The slope of the line $x = 5$ is: [1]
- (a) 0
 - (b) 1
 - (c) 5
 - (d) not defined
- Q17.** The distance of the point **(0, 0)** from the line $3x + 4y - 10 = 0$ is: [1]
- (a) 1
 - (b) 2
 - (c) 5
 - (d) 10
- Q18.** The length of the latus rectum of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ is: [1]
- (a) $\frac{9}{5}$
 - (b) $\frac{18}{5}$
 - (c) $\frac{6}{5}$
 - (d) $\frac{25}{3}$

Q19. Assertion (A): If A and B are disjoint sets, then $A \cap B = \emptyset$. Reason (R): Disjoint sets have no element in common. [1]

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is not the correct explanation of A
- (c) A is true, R is false
- (d) A is false, R is true

Q20. Assertion (A): For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the eccentricity $e > 1$. Reason (R): For a hyperbola $c > a$, where $c^2 = a^2 + b^2$. [1]

- (a) Both A and R are true and R is the correct explanation of A
- (b) Both A and R are true but R is not the correct explanation of A
- (c) A is true, R is false
- (d) A is false, R is true

SECTION B

Very Short Answer — 2 marks each

Q1. If $(x + 1, y - 2) = (3, 1)$, find the values of x and y . [2]

Q2. Find the value of $\cos 15^\circ$. [2]

Q3. Solve the inequality $3(x - 2) \leq 2(x + 1)$, $x \in \mathbb{R}$. [2]

Q4. Find n if ${}^nP_2 = 30$. [2]

Q5. Insert three arithmetic means between 1 and 9. [2]

SECTION C

Short Answer — 3 marks each

Q1. In a group of 50 people, 30 read Hindi newspapers, 20 read English newspapers and 10 read both. Find how many read neither Hindi nor English newspapers. [3]

Q2. Prove that $\cos 4x = 1 - 8 \sin^2 x \cos^2 x$. [3]

Q3. Express $\frac{1+i}{1-i}$ in the form $a + ib$ and find its modulus. [3]

Q4. Solve the system of inequalities $2x + 1 > x + 3$ and $x - 3 \leq 5$, and write the solution set. [3]

Q5. How many 3-digit even numbers with distinct digits can be formed from the digits 1, 2, 3, 4, 5? [3]

Q6. Find the coefficient of x^3 in the expansion of $(2x + 1)^5$. [3]

SECTION D

Long Answer — 5 marks each

Q1. Prove that $\tan x + \cot x = 2 \operatorname{cosec} 2x$. [5]

Q2. Solve the quadratic equation $x^2 + 3x + 5 = 0$ and express its roots in the form $a + ib$. [5]

Q3. Find the sum of the squares of the first n natural numbers $1^2 + 2^2 + 3^2 + \dots + n^2$, and hence find the sum of the squares of the first 10 natural numbers. [5]

- Q4.** Find the coordinates of the focus, the axis, the equation of the directrix and the length of the latus rectum of the parabola $x^2 = 16y$. [5]

SECTION E

Case-Study Based — 4 marks each

- Q1.** In a survey of 60 people, 25 like product A, 26 like product B, 26 like product C, 9 like A and B, 11 like B and C, 8 like A and C, and 8 like all three. (i) How many like A and B but not C? (ii) How many like only C? (iii) How many like at least one product? (iv) How many like none of the three? [4]
- Q2.** A culture of bacteria doubles every hour. Initially there are 50 bacteria, so the counts **50, 100, 200, ...** form a G.P. (i) Write the common ratio. (ii) How many bacteria are there after 5 hours (the 6th term)? (iii) Find the total number of bacteria counted at the start of each of the first 5 hours (the sum of the first 5 terms). [4]
- Q3.** A mobile-data plan charges a fixed rental plus a per-GB rate, so the monthly bill **y** (in Rs.) for **x** GB is **$y = 100 + 50x$** . (i) Write the slope of this line. (ii) Write the bill for 6 GB of data. (iii) Write the **y** -intercept and state what it represents. [4]

ANSWER KEY & MARKING SCHEME

SECTION A

Answers

- Q1. (c) 16
- Q2. (c) 16
- Q3. (c) $\mathbb{R} - \{-1, 1\}$
- Q4. (c) $[0, \infty)$
- Q5. (a) $\frac{1}{2}$
- Q6. (c) 2
- Q7. (c) i
- Q8. (a) 5
- Q9. (a) $x < 4$
- Q10. (c) 720
- Q11. (b) 35
- Q12. (b) 4th term
- Q13. (b) 34
- Q14. (a) 9
- Q15. (b) 3
- Q16. (d) not defined
- Q17. (b) 2
- Q18. (b) $\frac{18}{5}$
- Q19. (a) Both A and R are true and R is the correct explanation of A
- Q20. (a) Both A and R are true and R is the correct explanation of A

SECTION B

Answers

- Q1. $x + 1 = 3 \Rightarrow x = 2; y - 2 = 1 \Rightarrow y = 3.$
- Q2. $\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45 \cos 30 + \sin 45 \sin 30 = \frac{\sqrt{6} + \sqrt{2}}{4}.$
- Q3. $3x - 6 \leq 2x + 2 \Rightarrow x \leq 8.$ Solution $(-\infty, 8].$
- Q4. ${}^nP_2 = n(n-1) = 30 \Rightarrow n^2 - n - 30 = 0 \Rightarrow (n-6)(n+5) = 0 \Rightarrow n = 6.$
- Q5. Common difference $d = \frac{9-1}{4} = 2.$ The three means are 3, 5, 7.

SECTION C

Answers

- Q1. $n(H \cup E) = n(H) + n(E) - n(H \cap E) = 30 + 20 - 10 = 40.$ Neither = $50 - 40 = 10.$

Q2. $\cos 4x = 1 - 2 \sin^2 2x = 1 - 2(2 \sin x \cos x)^2 = 1 - 2 \cdot 4 \sin^2 x \cos^2 x = 1 - 8 \sin^2 x \cos^2 x$.

Q3. $\frac{1+i}{1-i} = \frac{(1+i)^2}{(1-i)(1+i)} = \frac{1+2i-1}{2} = \frac{2i}{2} = i = 0 + 1i$. Modulus = 1.

Q4. $2x + 1 > x + 3 \Rightarrow x > 2$; $x - 3 \leq 5 \Rightarrow x \leq 8$. Solution $(2, 8]$.

Q5. Units place must be even (2 or 4): 2 choices. Hundreds place: 4 remaining digits; tens place: 3 remaining. Total = $2 \times 4 \times 3 = 24$.

Q6. $T_{r+1} = {}^5C_r (2x)^{5-r} (1)^r$. For x^3 , $5 - r = 3 \Rightarrow r = 2$. Coefficient = ${}^5C_2 \cdot 2^3 = 10 \times 8 = 80$.

SECTION D

Answers

Q1. $\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x} = \frac{2}{2 \sin x \cos x} = \frac{2}{\sin 2x} = 2 \operatorname{cosec} 2x$.

Q2. $x = \frac{-3 \pm \sqrt{9-20}}{2} = \frac{-3 \pm \sqrt{-11}}{2} = -\frac{3}{2} \pm \frac{\sqrt{11}}{2}i$.

Q3. $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$. For $n = 10$: $\frac{10 \cdot 11 \cdot 21}{6} = 385$.

Q4. Compare $x^2 = 16y$ with $x^2 = 4ay \Rightarrow a = 4$. Focus $(0, 4)$; axis is the y -axis, $x = 0$; directrix $y = -4$; latus rectum = 16.

SECTION E

Answers

Q1. Using a Venn diagram with $n(A \cap B \cap C) = 8$: (i) A and B but not C = $9 - 8 = 1$. (ii) Only C = $26 - (8 + 3 + 0) = 15$. (iii) At least one = $25 + 26 + 26 - 9 - 11 - 8 + 8 = 57$. (iv) None = $60 - 57 = 3$.

Q2. (i) Common ratio $r = 2$. (ii) 6th term = $50 \cdot 2^5 = 1600$ bacteria. (iii) Sum of first 5 terms = $50 \cdot \frac{2^5 - 1}{2 - 1} = 50 \times 31 = 1550$.

Q3. (i) Slope = 50 (Rs. per GB). (ii) Bill for 6 GB = $100 + 50(6) = 400$. (iii) y -intercept = 100; it is the fixed monthly rental charged even for 0 GB.