

**MID-TERM EXAMINATION — Class 11 Mathematics (Set B)****Chapters:** Sets, Relations & Functions, Trigonometric Functions, Complex Numbers & Quadratic Equations, Linear Inequalities, Permutations & Combinations, Binomial Theorem, Sequences & Series, Straight Lines, Conic Sections**Max
Marks:
80**

Name: _____

Time: 3 Hours**General Instructions:** All questions are compulsory. The paper has five sections — A (20×1), B (5×2), C (6×3), D (4×5) and E (3×4, case-studies). Draw neat figures where required. Use of a calculator is not permitted.**SECTION A***Multiple Choice / Assertion-Reason — 1 mark each*

- Q1.** If $A = \{x : x \in \mathbb{N}, x \leq 5\}$, then the number of subsets of A is: [1]
(a) 10
(b) 25
(c) 32
(d) 64
- Q2.** If $A \subseteq B$ and $B \subseteq A$, then: [1]
(a) $A \subset B$
(b) $A = B$
(c) $A = \emptyset$
(d) $B = \emptyset$
- Q3.** If $n(A) = 2$ and $n(B) = 3$, then $n(A \times B)$ is: [1]
(a) 5
(b) 6
(c) 8
(d) 9
- Q4.** The domain of the function $f(x) = \sqrt{9 - x^2}$ is: [1]
(a) $(-3, 3)$
(b) $[-3, 3]$
(c) $[0, 3]$
(d) $\mathbb{R}$
- Q5.** The value of $\cos 0^\circ + \sin 90^\circ$ is: [1]
(a) 0
(b) 1
(c) 2
(d) $\frac{1}{2}$
- Q6.** If $\sin \theta = \frac{3}{5}$ and θ is acute, then $\tan \theta$ equals: [1]
(a) $\frac{3}{4}$
(b) $\frac{4}{3}$
(c) $\frac{4}{5}$
(d) $\frac{5}{3}$
- Q7.** The value of i^{16} is: [1]
(a) i
(b) $-i$
(c) 1
(d) -1

- Q8.** The conjugate of the complex number $2 - 3i$ is: [1]
(a) $-2 + 3i$
(b) $2 + 3i$
(c) $-2 - 3i$
(d) $3 + 2i$
- Q9.** The solution set of $|2x - 1| \leq 3$ is: [1]
(a) $[-1, 2]$
(b) $(-1, 2)$
(c) $[1, 2]$
(d) $[-2, 1]$
- Q10.** The number of arrangements of the letters of the word SUNDAY is: [1]
(a) 120
(b) 360
(c) 720
(d) 5040
- Q11.** The value of ${}^{10}C_2$ is: [1]
(a) 20
(b) 45
(c) 90
(d) 100
- Q12.** The sum of the coefficients in the expansion of $(1 + x)^5$ is: [1]
(a) 16
(b) 25
(c) 32
(d) 64
- Q13.** The 5th term of the A.P. $3, 5, 7, \dots$ is: [1]
(a) 9
(b) 11
(c) 13
(d) 15
- Q14.** The arithmetic mean of 4 and 16 is: [1]
(a) 8
(b) 10
(c) 12
(d) 20
- Q15.** The 4th term of the G.P. $3, 6, 12, \dots$ is: [1]
(a) 18
(b) 24
(c) 36
(d) 48
- Q16.** The slope of the line joining the points $(1, 2)$ and $(3, 6)$ is: [1]
(a) 1
(b) 2
(c) 3
(d) $\frac{1}{2}$
- Q17.** The x -intercept of the line $2x + 3y = 6$ is: [1]
(a) 2
(b) 3
(c) 6
(d) -3

- Q18.** The radius of the circle $x^2 + y^2 = 49$ is: [1]
 (a) 7
 (b) 49
 (c) 14
 (d) $\sqrt{7}$
- Q19.** Assertion (A): For any set A , $A \cup A' = U$. Reason (R): A and A' have no element in common. [1]
 (a) Both A and R are true and R is the correct explanation of A
 (b) Both A and R are true but R is not the correct explanation of A
 (c) A is true, R is false
 (d) A is false, R is true
- Q20.** Assertion (A): The eccentricity of an ellipse satisfies $0 < e < 1$. Reason (R): For an ellipse $c < a$, where c is the distance of a focus from the centre and a is the semi-major axis. [1]
 (a) Both A and R are true and R is the correct explanation of A
 (b) Both A and R are true but R is not the correct explanation of A
 (c) A is true, R is false
 (d) A is false, R is true

SECTION B

Very Short Answer — 2 marks each

- Q1.** If $A = \{1, 2, 3\}$ and $B = \{3, 4\}$, write $A \times B$ and state $n(A \times B)$. [2]
- Q2.** Find the value of $\sin 75^\circ$. [2]
- Q3.** Solve the inequality $7x + 3 < 5x + 9$, $x \in \mathbb{R}$, and represent it on the number line. [2]
- Q4.** Find n if ${}^nC_2 = 15$. [2]
- Q5.** Find the 20th term of the A.P. 5, 8, 11, 14, ... [2]

SECTION C

Short Answer — 3 marks each

- Q1.** In a class of 35 students, 24 like tea and 20 like coffee. If each student likes at least one of the two drinks, find how many students like both tea and coffee. [3]
- Q2.** Prove that $\cos 3x = 4\cos^3 x - 3\cos x$. [3]
- Q3.** Find the multiplicative inverse of the complex number $3 + 4i$. [3]
- Q4.** Solve the inequality $-8 \leq 5x - 3 < 7$, $x \in \mathbb{R}$, and represent the solution on the number line. [3]
- Q5.** How many words, with or without meaning, of 4 letters can be formed from the letters of the word MONDAY, if repetition is not allowed? [3]
- Q6.** Find the term independent of x in the expansion of $\left(x + \frac{1}{x}\right)^6$. [3]

SECTION D

Long Answer — 5 marks each

- Q1.** Prove that $\frac{\sin 7x + \sin 5x}{\cos 7x + \cos 5x} = \tan 6x$. [5]
- Q2.** Solve the quadratic equation $2x^2 + x + 1 = 0$ and express its roots in the form $a + ib$. [5]
- Q3.** Find the sum of the first n terms of the series $3 + 7 + 11 + 15 + \dots$ and hence find the sum of the first 15 terms. [5]
- Q4.** Find the equation of the ellipse whose vertices are $(\pm 5, 0)$ and foci are $(\pm 4, 0)$. Also find its eccentricity and the length of its latus rectum. [5]

SECTION E

Case-Study Based — 4 marks each

- Q1.** In a survey of 100 students, it was found that 60 play cricket, 45 play football and 25 play both games. (i) How many play cricket only? (ii) How many play football only? (iii) How many play at least one of the two games? (iv) How many play neither game? [4]
- Q2.** A person is offered a starting salary of Rs. 20000 per month, with an annual increment of Rs. 1500 per month. The monthly salaries in successive years form an A.P. (i) Write the common difference of the A.P. (ii) What will the person's monthly salary be in the 6th year? (iii) Find the total of the monthly salaries over the first 5 years (i.e. the sum of the first 5 terms). [4]
- Q3.** The arch of a bridge is in the shape of a parabola $x^2 = 8y$ (vertex at the origin, at the top of the arch axis pointing up). (i) Write the coordinates of the focus. (ii) Write the length of the latus rectum. (iii) Write the equation of the directrix. [4]

ANSWER KEY & MARKING SCHEME

SECTION A

Answers

- Q1. (c) 32
- Q2. (b) $A = B$
- Q3. (b) 6
- Q4. (b) $[-3, 3]$
- Q5. (c) 2
- Q6. (a) $\frac{3}{4}$
- Q7. (c) 1
- Q8. (b) $2 + 3i$
- Q9. (a) $[-1, 2]$
- Q10. (c) 720
- Q11. (b) 45
- Q12. (c) 32
- Q13. (b) 11
- Q14. (b) 10
- Q15. (b) 24
- Q16. (b) 2
- Q17. (b) 3
- Q18. (a) 7
- Q19. (b) Both A and R are true but R is not the correct explanation of A
- Q20. (a) Both A and R are true and R is the correct explanation of A

SECTION B

Answers

- Q1. $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}; n(A \times B) = 6.$
- Q2. $\sin 75^\circ = \sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}.$
- Q3. $7x + 3 < 5x + 9 \Rightarrow 2x < 6 \Rightarrow x < 3.$ Solution $(-\infty, 3)$; open circle at 3, arrow to the left.
- Q4. ${}^nC_2 = \frac{n(n-1)}{2} = 15 \Rightarrow n^2 - n - 30 = 0 \Rightarrow (n-6)(n+5) = 0 \Rightarrow n = 6.$
- Q5. $a_{20} = a + 19d = 5 + 19(3) = 5 + 57 = 62.$

SECTION C

Answers

- Q1. $n(T \cap C) = n(T) + n(C) - n(T \cup C) = 24 + 20 - 35 = 9.$ So 9 students like both.
- Q2. $\cos 3x = \cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x = (2 \cos^2 x - 1) \cos x - 2 \sin^2 x \cos x = 2 \cos^3 x - \cos x - 2(1 - \cos^2 x) \cos x = 4 \cos^3 x - 3 \cos x.$
- Q3. Inverse $= \frac{1}{3+4i} = \frac{3-4i}{(3+4i)(3-4i)} = \frac{3-4i}{9+16} = \frac{3}{25} - \frac{4}{25}i.$
- Q4. $-8 \leq 5x - 3 \Rightarrow -5 \leq 5x \Rightarrow x \geq -1;$ and $5x - 3 < 7 \Rightarrow x < 2.$ Solution $[-1, 2).$
- Q5. Number of 4-letter words $= {}^6P_4 = \frac{6!}{2!} = 360.$
- Q6. $T_{r+1} = {}^6C_r x^{6-r} x^{-r} = {}^6C_r x^{6-2r}.$ Independent of x when $6 - 2r = 0 \Rightarrow r = 3.$ Term $= {}^6C_3 = 20.$

SECTION D

Answers

- Q1. Numerator $= \sin 7x + \sin 5x = 2 \sin 6x \cos x;$ Denominator $= \cos 7x + \cos 5x = 2 \cos 6x \cos x.$ Ratio $= \frac{2 \sin 6x \cos x}{2 \cos 6x \cos x} = \tan 6x.$
- Q2. $x = \frac{-1 \pm \sqrt{1-8}}{4} = \frac{-1 \pm \sqrt{-7}}{4} = -\frac{1}{4} \pm \frac{\sqrt{7}}{4}i.$ Roots: $-\frac{1}{4} + \frac{\sqrt{7}}{4}i$ and $-\frac{1}{4} - \frac{\sqrt{7}}{4}i.$
- Q3. A.P. with $a = 3, d = 4.$ $S_n = \frac{n}{2} [2(3) + (n-1)4] = \frac{n}{2} (4n+2) = n(2n+1).$ $S_{15} = 15(31) = 465.$
- Q4. $a = 5, c = 4 \Rightarrow b^2 = a^2 - c^2 = 25 - 16 = 9.$ Ellipse: $\frac{x^2}{25} + \frac{y^2}{9} = 1.$ Eccentricity $e = \frac{c}{a} = \frac{4}{5};$ latus rectum $= \frac{2b^2}{a} = \frac{18}{5}.$

- Q1. (i) Cricket only = $60 - 25 = 35$. (ii) Football only = $45 - 25 = 20$. (iii) At least one = $60 + 45 - 25 = 80$. (iv) Neither = $100 - 80 = 20$.
- Q2. (i) Common difference $d = 1500$. (ii) 6th-year salary = $20000 + 5(1500) = 27500$. (iii) Sum of first 5 terms
 $= \frac{5}{2}[2(20000) + 4(1500)] = \frac{5}{2}(46000) = 115000$.
- Q3. Compare $x^2 = 8y$ with $x^2 = 4ay \Rightarrow 4a = 8 \Rightarrow a = 2$. (i) Focus $(0, 2)$. (ii) Latus rectum = 8. (iii) Directrix $y = -2$.