

**Mid-Term Examination — Mathematics**

Class XI • CBSE • Mid-Term Examination

Name: \_\_\_\_\_

Roll No.: \_\_\_\_\_

Max. Marks: 80

Time: 3 Hours

**General Instructions:** (i) This question paper contains 38 questions. All questions are compulsory. (ii) The paper is divided into five sections — A, B, C, D and E. (iii) Section A has 20 questions of 1 mark each (MCQ and Assertion-Reason). (iv) Section B has 5 questions of 2 marks each; Section C has 6 questions of 3 marks each; Section D has 4 questions of 5 marks each; Section E has 3 case-study questions of 4 marks each. (v) Use of a calculator is **not** permitted.

**Section A (Multiple Choice & Assertion-Reason — 20 × 1 = 20 marks)***All questions are compulsory. Each carries 1 mark.*

- If a set  $A$  has 3 elements, the number of subsets of  $A$  is: (a) 3 (b) 6 (c) 8 (d) 9
- If  $n(A) = 3$  and  $n(B) = 6$  and  $A \subseteq B$ , then the minimum number of elements in  $A \cup B$  is: (a) 3 (b) 6 (c) 9 (d) 18
- The domain of the function  $f(x) = \frac{1}{\sqrt{x-5}}$  is: (a)  $[5, \infty)$  (b)  $(5, \infty)$  (c)  $(-\infty, 5)$  (d)  $\mathbb{R} - \{5\}$
- The range of the function  $f(x) = |x - 1|$  is: (a)  $\mathbb{R}$  (b)  $(0, \infty)$  (c)  $[0, \infty)$  (d)  $[1, \infty)$
- The value of  $\sin 765^\circ$  is: (a)  $\frac{1}{2}$  (b)  $\frac{1}{\sqrt{2}}$  (c)  $\frac{\sqrt{3}}{2}$  (d) 1
- If  $\tan x = \frac{3}{4}$  and  $x$  lies in the third quadrant, then  $\cos x$  equals: (a)  $\frac{4}{5}$  (b)  $-\frac{4}{5}$  (c)  $\frac{3}{5}$  (d)  $-\frac{3}{5}$
- The value of  $i^{-39}$  is: (a) 1 (b)  $-1$  (c)  $i$  (d)  $-i$
- The modulus of the complex number  $1 + i$  is: (a) 1 (b)  $\sqrt{2}$  (c) 2 (d) 0
- The solution set of  $|x| < 3$  is: (a)  $x < 3$  (b)  $x > -3$  (c)  $-3 < x < 3$  (d)  $x \leq 3$
- The number of arrangements of the letters of the word DELHI is: (a) 24 (b) 60 (c) 120 (d) 720
- The number of diagonals of a hexagon is: (a) 6 (b) 9 (c) 12 (d) 18
- The number of terms in the expansion of  $(x + y)^{10}$  is: (a) 9 (b) 10 (c) 11 (d) 12
- The middle term in the expansion of  $(x + y)^8$  is the: (a) 4th term (b) 5th term (c) 6th term (d) 3rd term
- The 10th term of the A.P. 2, 7, 12, ... is: (a) 45 (b) 47 (c) 52 (d) 42
- The geometric mean of 4 and 9 is: (a) 6 (b)  $\frac{13}{2}$  (c) 36 (d) 5
- The slope of the line  $3x + 4y = 12$  is: (a)  $\frac{3}{4}$  (b)  $-\frac{3}{4}$  (c)  $\frac{4}{3}$  (d) 3
- The distance between the points  $(0, 0)$  and  $(3, 4)$  is: (a) 4 (b) 5 (c) 7 (d)  $\sqrt{7}$
- For the parabola  $y^2 = 12x$ , the length of the latus rectum is: (a) 3 (b) 6 (c) 12 (d) 24
- Assertion (A): The empty set  $\emptyset$  is a subset of every set. Reason (R): A set is a subset of itself. (a) Both A and R are true and R is the correct explanation of A (b) Both A and R are true but R is not the correct explanation of A (c) A is true, R is false (d) A is false, R is true
- Assertion (A): For the parabola  $y^2 = 4ax$ , the focus is  $(a, 0)$ . Reason (R): The directrix of  $y^2 = 4ax$  is  $x = -a$ . (a) Both A and R are true and R is the correct explanation of A (b) Both A and R are true but R is not the correct explanation of A (c) A is true, R is false (d) A is false, R is true

### Section B (Very Short Answer — 5 × 2 = 10 marks)

Each question carries 2 marks.

1. If  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $A = \{2, 4, 6, 8\}$  and  $B = \{1, 3, 5, 7, 9\}$ , find  $(A \cup B)'$ .
2. Find the value of  $\cos(-1710^\circ)$ .
3. Solve the inequality  $3x + 2 < 2x + 7$ , where  $x \in \mathbb{R}$ , and represent the solution on the number line.
4. If  ${}^nP_2 = 20$ , find the value of  $n$ .
5. Find the sum of the first 20 terms of the A.P. 1, 4, 7, 10, ...

### Section C (Short Answer — 6 × 3 = 18 marks)

Each question carries 3 marks.

1. In a group of 60 people, 27 like cold drinks and 42 like coffee. If each person likes at least one of the two drinks, find how many people like both cold drinks and coffee.
2. Prove that  $\sin 3x = 3 \sin x - 4 \sin^3 x$ .
3. Find the modulus and the argument of the complex number  $z = -1 + i\sqrt{3}$ .
4. Solve the system of inequalities  $5(2x - 7) - 3(2x + 3) \leq 0$  and  $2x + 19 \leq 6x + 47$ , and represent the solution on the number line.
5. How many 4-digit numbers can be formed using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 if no digit is repeated?
6. Find the coefficient of  $x^5$  in the expansion of  $(x + 3)^8$ .

### Section D (Long Answer — 4 × 5 = 20 marks)

Each question carries 5 marks.

1. Prove that  $\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$ .
2. Solve the quadratic equation  $x^2 + x + 1 = 0$  and express its roots in the form  $a + ib$ . Also state the sum and product of the roots.
3. Find the sum to  $n$  terms of the series  $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots$
4. Find the coordinates of the focus, the axis, the equation of the directrix and the length of the latus rectum of the parabola  $y^2 = 12x$ . Also find the equation of the line passing through the focus and parallel to the directrix.

### Section E (Case-Study Based — 3 × 4 = 12 marks)

Each case study carries 4 marks. Read the passage and answer the parts that follow.

1. A school surveyed 100 students about the languages they study. It was found that 60 study French, 50 study Spanish and 20 study both French and Spanish. (i) How many students study French only? (ii) How many study Spanish only? (iii) How many study at least one of the two languages? (iv) How many study neither language?
2. A ball is dropped from a height of 32 m. Each time it strikes the ground it rebounds to a height that is half of the previous height, forming a geometric progression 32, 16, 8, ... (i) Write the common ratio of the G.P. (ii) Find the height of the ball after the 5th rebound. (iii) Find the total distance covered by the ball before it comes to rest (consider the total up-and-down distance).
3. A suspension bridge cable hangs in the shape of a parabola with equation  $x^2 = 20y$  (origin at the lowest point of the cable). (i) Write the coordinates of the focus of the parabola. (ii) Write the length of the latus rectum. (iii) Write the equation of the directrix.

## Answer Key & Marking Scheme

For teacher use • Kumar Classes

### Section A

1. (c) 8
2. (b) 6
3. (b)  $(5, \infty)$
4. (c)  $[0, \infty)$
5. (b)  $\frac{1}{\sqrt{2}}$
6. (b)  $-\frac{4}{5}$
7. (c)  $i$
8. (b)  $\sqrt{2}$
9. (c)  $-3 < x < 3$
10. (c) 120
11. (b) 9
12. (c) 11
13. (b) 5th term
14. (b)  $4\bar{7}$
15. (a) 6
16. (b)  $-\frac{3}{4}$
17. (b) 5
18. (c) 12
19. (b) Both A and R are true but R is not the correct explanation of A
20. (b) Both A and R are true but R is not the correct explanation of A

### Section B

1.  $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ , so  $(A \cup B)' = U - (A \cup B) = \{10\}$ .
2.  $\cos(-1710^\circ) = \cos(1710^\circ) = \cos(1710 - 4 \times 360)^\circ = \cos(270^\circ) = 0$ .
3.  $3x + 2 < 2x + 7 \Rightarrow x < 5$ . Solution set  $(-\infty, 5)$ : open circle at 5, arrow to the left.
4.  ${}^nP_2 = n(n-1) = 20 \Rightarrow n^2 - n - 20 = 0 \Rightarrow (n-5)(n+4) = 0 \Rightarrow n = 5$  (reject  $n = -4$ ).
5.  $S_{20} = \frac{20}{2} [2(1) + (20-1)(3)] = 10(2+57) = 10 \times 59 = 590$ .

### Section C

1.  $n(C \cup F) = 60$ ,  $n(C) = 27$ ,  $n(F) = 42$ .  $n(C \cap F) = n(C) + n(F) - n(C \cup F) = 27 + 42 - 60 = 9$ . So 9 people like both.
2.  $\sin 3x = \sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x = 2 \sin x \cos^2 x + (1 - 2 \sin^2 x) \sin x = 2 \sin x (1 - \sin^2 x) + \sin x - 2 \sin^3 x$
3.  $|z| = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{4} = 2$ .  $z$  lies in the second quadrant,  $\tan \theta = \frac{\sqrt{3}}{-1}$ , so argument  $= \pi - \frac{\pi}{3} = \frac{2\pi}{3} = 120^\circ$ .

4. First:  $10x - 35 - 6x - 9 \leq 0 \Rightarrow 4x \leq 44 \Rightarrow x \leq 11$ . Second:  $2x + 19 \leq 6x + 47 \Rightarrow -28 \leq 4x \Rightarrow x \geq -7$ . Common solution  $[-7, 11]$ .
5. The thousands place: 9 choices, then 8, 7, 6. Total  $= 9 \times 8 \times 7 \times 6 = 3024$  numbers.
6. General term  $T_{r+1} = {}^8C_r x^{8-r} 3^r$ . For  $x^5$ ,  $8 - r = 5 \Rightarrow r = 3$ . Coefficient  $= {}^8C_3 \cdot 3^3 = 56 \times 27 = 1512$ .

### Section D

1. Numerator  $= (\cos 4x + \cos 2x) + \cos 3x = 2 \cos 3x \cos x + \cos 3x = \cos 3x(2 \cos x + 1)$ . Denominator  $= (\sin 4x + \sin 2x) + \sin 3x = 2 \sin 3x \cos x + \sin 3x = \sin 3x(2 \cos x + 1)$ . Ratio  $= \frac{\cos 3x(2 \cos x + 1)}{\sin 3x(2 \cos x + 1)} = \cot 3x$ .
2. Using the quadratic formula,  $x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$ . Roots:  $-\frac{1}{2} + \frac{\sqrt{3}}{2}i$  and  $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$ . Sum  $= -1$ . Product  $= 1$ .
3. The  $k$ th term  $a_k = k(k+1) = k^2 + k$ .  

$$S_n = \sum_{k=1}^n (k^2 + k) = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} = \frac{n(n+1)}{6} [(2n+1) + 3] = \frac{n(n+1)(2n+4)}{6} = \frac{n(n+1)(n+2)}{3}.$$
4. Compare with  $y^2 = 4ax$ :  $4a = 12 \Rightarrow a = 3$ . Focus  $(3, 0)$ ; axis is the  $x$ -axis,  $y = 0$ ; directrix  $x = -3$ ; length of latus rectum  $= 4a = 12$ . Line through focus parallel to the directrix (a vertical line):  $x = 3$ .

### Section E

1. (i) French only  $= n(F') - n(F' \cap S) = 60 - 20 = 40$ . (ii) Spanish only  $= 50 - 20 = 30$ . (iii) At least one  $= 60 + 50 - 20 = 90$ . (iv) Neither  $= 100 - 90 = 10$ .
2. (i) Common ratio  $r = \frac{1}{2}$ . (ii) Heights after rebounds form **16, 8, 4, 2, 1**; height after 5th rebound  $= 32\left(\frac{1}{2}\right)^5 = 1$  m. (iii) Total distance  $= 32 + 2(16 + 8 + 4 + \dots) = 32 + 2 \times \frac{16}{1 - \frac{1}{2}} = 32 + 2 \times 32 = 96$  m.
3. Compare  $x^2 = 20y$  with  $x^2 = 4ay$ :  $4a = 20 \Rightarrow a = 5$ . (i) Focus  $(0, 5)$ . (ii) Length of latus rectum  $= 20$ . (iii) Directrix  $y = -5$ .