

**MID-TERM EXAMINATION — Class 10 Mathematics (Set 3)**

Chapters: GST, Banking, Linear Inequations, Quadratic Equations, Ratio & Proportion, Factorisation, Matrices, Arithmetic Progression, Geometric Progression, Reflection, Mid-point & Section Formula, Equation of a Line, Similarity, Circles, Trigonometric Identities, Probability

Max Marks: 80

Name: _____

Time: 2½ Hours

General Instructions: Answers to this paper must be written on the paper provided separately. You will **not** be allowed to write during the first 15 minutes (reading time). Attempt **all** questions from Section A and **any four** questions from Section B. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks. Mathematical tables and squared paper are provided.

SECTION A**40 Marks — Attempt ALL questions***Attempt all questions from this Section.*

- Q1.** (i) A manufacturer sells a television to a dealer for Rs. 18000, and the dealer sells it to a consumer at a profit of Rs. 2500. If the GST rate is 18% and all transactions are intra-state, find the GST paid by the dealer to the government. [3]
- (ii) Ravi deposits Rs. 1200 per month in a recurring deposit account for 18 months at 8% per annum. Find the maturity value of the account. [3]
- (iii) If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$, find the matrix $2A - 3B$. [4]
- Q2.** (i) Solve the inequation $3(2x - 1) \geq 2(2x + 3)$, where $x \in \mathbf{R}$, and represent the solution on a number line. [3]
- (ii) Solve the quadratic equation $x^2 - 4x - 8 = 0$, giving your answer correct to two decimal places. [4]
- (iii) If $a : b = 5 : 3$, find the value of $(7a - 3b) : (2a + b)$. [3]
- Q3.** (i) Show that $(x - 1)$ and $(x + 2)$ are factors of $x^3 + 2x^2 - x - 2$, and factorise it completely. [3]
- (ii) The 7th term of an A.P. is 20 and its 13th term is 38. Find the A.P. and the sum of its first 15 terms. [4]
- (iii) Find the 8th term of the G.P. 256, 128, 64, ... [3]
- Q4.** (i) The point $A(4, 5)$ is reflected in the x -axis to A' , and A' is reflected in the y -axis to A'' . Write the coordinates of A' and A'' , and name the single transformation that maps A to A'' . [4]
- (ii) Find the ratio in which the point $(-4, 6)$ divides the line segment joining $A(-6, 10)$ and $B(3, -8)$. [3]
- (iii) Prove that $\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \operatorname{cosec} \theta - \cot \theta$. [3]

SECTION B**40 Marks — Attempt any FOUR questions***Attempt any four questions from this Section.*

- Q5.** (i) Find the equation of the line passing through the points $(0, -2)$ and $(3, 4)$. [3]
(ii) Find the equation of the line perpendicular to $x - 2y = 6$ and passing through the point $(2, -1)$. [3]
(iii) The line $3x + 4y = 12$ meets the x -axis at A and the y -axis at B . Find the coordinates of A and B and the length AB . [4]
- Q6.** (i) In $\triangle ABC$, right-angled at B , $BD \perp AC$. If $AB = 6$ cm and $BC = 8$ cm, find AC and BD . [4]
(ii) The areas of two similar triangles are in the ratio $16 : 25$. If the perimeter of the smaller triangle is 48 cm, find the perimeter of the larger triangle. [3]
(iii) In $\triangle ABC$, $DE \parallel BC$ with $AD = 4$ cm and $AB = 10$ cm. If the area of $\triangle ABC$ is 75 cm^2 , find the area of $\triangle ADE$. [3]
- Q7.** (i) AB is a diameter of a circle and C is a point on the circle. If $\angle BAC = 40^\circ$, find $\angle ACB$ and $\angle ABC$. [4]
(ii) A tangent drawn from an external point to a circle has length 15 cm, and the external point is 17 cm from the centre. Find the radius of the circle. [3]
(iii) Two chords AB and CD of a circle intersect internally at P . If $PA = 6$ cm, $PB = 4$ cm and $PC = 3$ cm, find PD . [3]
- Q8.** (i) A card is drawn at random from a well-shuffled pack of 52 playing cards. Find the probability that it is (a) a king, (b) a red card. [3]
(ii) An article is marked at Rs. 1500 and sold at a discount of 20%. If GST is charged at 18%, find the total amount paid by the customer. [3]
(iii) The sum of a number and its reciprocal is $\frac{26}{5}$. Form a quadratic equation and find the number. [4]
- Q9.** (i) Find the sum of the first 20 terms of the A.P. $7, 11, 15, \dots$ [3]
(ii) The 4th term of a G.P. is 24 and its 6th term is 96. Find the first term and the common ratio (taking it positive). [3]
(iii) Find three numbers in G.P. whose sum is 21 and the sum of whose squares is 189. [4]
- Q10.** (i) If $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, find A^2 . [4]
(ii) The monthly incomes of A and B are in the ratio $5 : 4$ and their monthly expenditures are in the ratio $3 : 2$. If each saves Rs. 6000 per month, find the monthly income of A. [3]
(iii) Anita deposits Rs. 150 per month in a recurring deposit account for 8 months and receives Rs. 1236 at maturity. Find the rate of interest per annum. [3]
- Q11.** (i) Prove that $\frac{\sin \theta}{\sec \theta + 1} + \frac{\sin \theta}{\sec \theta - 1} = 2 \cot \theta$. [4]
(ii) The point $(-3, -4)$ is reflected in the y -axis to P' , and P' is reflected in the x -axis to P'' . Write the coordinates of P' and P'' . [3]
(iii) In a cyclic quadrilateral $PQRS$, $\angle P = 70^\circ$ and $\angle Q = 80^\circ$. Find $\angle R$ and $\angle S$. [3]

ANSWER KEY & MARKING SCHEME

SECTION A

Answers

- Q1.** (i) Value addition by dealer = **2500**; GST paid to government = **18%** of **2500** = **Rs. 450**. (ii) $n = 18$;
 $I = 1200 \cdot \frac{18 \times 19}{24} \cdot \frac{8}{100} = 1200 \times 14.25 \times 0.08 = \text{Rs. } 1368$; maturity = $1200 \times 18 + 1368 = \text{Rs. } 22968$.
 (iii) $2A = \begin{bmatrix} 4 & 6 \\ 2 & -8 \end{bmatrix}$, $3B = \begin{bmatrix} 3 & 0 \\ 6 & -3 \end{bmatrix}$; $2A - 3B = \begin{bmatrix} 1 & 6 \\ -4 & -5 \end{bmatrix}$.
- Q2.** (i) $6x - 3 \geq 4x + 6 \Rightarrow 2x \geq 9 \Rightarrow x \geq \frac{9}{2}$. Solution $\left[\frac{9}{2}, \infty\right)$. (ii)
 $x = \frac{4 \pm \sqrt{16 + 32}}{2} = \frac{4 \pm \sqrt{48}}{2} = 2 \pm 2\sqrt{3} \Rightarrow x \approx 5.46$ or $x \approx -1.46$. (iii) With $a = 5, b = 3$:
 $(7a - 3b) : (2a + b) = (35 - 9) : (10 + 3) = 26 : 13 = 2 : 1$.
- Q3.** (i) $f(1) = 1 + 2 - 1 - 2 = 0$ and $f(-2) = -8 + 8 + 2 - 2 = 0$, so both are factors;
 $x^3 + 2x^2 - x - 2 = (x - 1)(x + 1)(x + 2)$. (ii) $a + 6d = 20$, $a + 12d = 38 \Rightarrow 6d = 18$, $d = 3$, $a = 2$.
 A.P.: $2, 5, 8, \dots$; $S_{15} = \frac{15}{2}[4 + 14(3)] = \frac{15}{2}(46) = 345$. (iii) $a = 256$, $r = \frac{1}{2}$; $T_8 = 256\left(\frac{1}{2}\right)^7 = \frac{256}{128} = 2$.
- Q4.** (i) $A' = (4, -5)$; $A'' = (-4, -5)$; single transformation is reflection in the origin. (ii) Let the ratio be $k : 1$.
 $\frac{3k - 6}{k + 1} = -4 \Rightarrow 3k - 6 = -4k - 4 \Rightarrow 7k = 2 \Rightarrow k = \frac{2}{7}$; ratio = $2 : 7$ (checking y : $\frac{2(-8) + 7(10)}{9} = 6$).
 (iii) $\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \sqrt{\frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}} = \frac{1 - \cos \theta}{\sin \theta} = \operatorname{cosec} \theta - \cot \theta$.

SECTION B

Answers

- Q5.** (i) Slope = $\frac{4 - (-2)}{3 - 0} = 2$; equation $y = 2x - 2$, i.e. $2x - y - 2 = 0$. (ii) Slope of $x - 2y = 6$ is $\frac{1}{2}$, so required slope = -2 ; $y + 1 = -2(x - 2) \Rightarrow 2x + y - 3 = 0$. (iii) $A = (4, 0)$ (put $y = 0$), $B = (0, 3)$ (put $x = 0$);
 $AB = \sqrt{4^2 + 3^2} = 5$ units.
- Q6.** (i) $AC = \sqrt{6^2 + 8^2} = 10$ cm; $BD = \frac{AB \cdot BC}{AC} = \frac{48}{10} = 4.8$ cm. (ii) Ratio of sides = $\sqrt{16 : 25} = 4 : 5$;
 perimeter of larger = $48 \times \frac{5}{4} = 60$ cm. (iii) $\frac{AD}{AB} = \frac{4}{10} = \frac{2}{5}$, ratio of areas = $\frac{4}{25}$; area
 $\triangle ADE = 75 \times \frac{4}{25} = 12 \text{ cm}^2$.
- Q7.** (i) Angle in a semicircle $\angle ACB = 90^\circ$; $\angle ABC = 180^\circ - 90^\circ - 40^\circ = 50^\circ$. (ii) Radius
 $= \sqrt{17^2 - 15^2} = \sqrt{289 - 225} = 8$ cm. (iii) $PA \cdot PB = PC \cdot PD \Rightarrow 6 \times 4 = 3 \times PD \Rightarrow PD = 8$ cm.
- Q8.** (i) $P(\text{king}) = \frac{4}{52} = \frac{1}{13}$; $P(\text{red}) = \frac{26}{52} = \frac{1}{2}$. (ii) Selling price = $1500 \times 0.8 = 1200$; amount
 $= 1200 + 18\% \text{ of } 1200 = 1200 \times 1.18 = \text{Rs. } 1416$. (iii)
 $x + \frac{1}{x} = \frac{26}{5} \Rightarrow 5x^2 - 26x + 5 = 0 \Rightarrow (5x - 1)(x - 5) = 0 \Rightarrow x = 5 \text{ or } \frac{1}{5}$.
- Q9.** (i) $a = 7$, $d = 4$; $S_{20} = \frac{20}{2}[14 + 19(4)] = 10(90) = 900$. (ii) $ar^3 = 24$, $ar^5 = 96 \Rightarrow r^2 = 4$, $r = 2$, $a = 3$.
 (iii) The numbers are **3, 6, 12** (with $r = 2$): sum = **21** and sum of squares = **9 + 36 + 144 = 189**.

- Q10.** (i) $A^2 = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 0 & 9 \end{bmatrix}$. (ii) Let incomes = $5x, 4x$ and expenditures = $3y, 2y$: $5x - 3y = 6000$ and $4x - 2y = 6000$. Solving gives $x = 3000$, so income of A = $5x = \text{Rs. } 15000$. (iii) Deposits = 1200 , interest = 36 ; $I = 150 \cdot \frac{8 \times 9}{24} \cdot \frac{r}{100} = 4.5r = 36 \Rightarrow r = 8\%$.
- Q11.** (i) $\text{LHS} = \sin \theta \left[\frac{(\sec \theta - 1) + (\sec \theta + 1)}{\sec^2 \theta - 1} \right] = \sin \theta \cdot \frac{2 \sec \theta}{\tan^2 \theta} = 2 \sin \theta \cdot \frac{1}{\cos \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{2 \cos \theta}{\sin \theta} = 2 \cot \theta$.
- (ii) $P' = (3, -4)$; $P'' = (3, 4)$. (iii) $\angle R = 180^\circ - \angle P = 110^\circ$; $\angle S = 180^\circ - \angle Q = 100^\circ$.