

**MID-TERM EXAMINATION — Class 10 Mathematics (Set 1)**

Chapters: GST, Banking, Linear Inequations, Quadratic Equations, Ratio & Proportion, Factorisation, Matrices, Arithmetic Progression, Geometric Progression, Reflection, Mid-point & Section Formula, Equation of a Line, Similarity, Circles, Trigonometric Identities, Probability

Max Marks: 80

Name: _____

Time: 2½ Hours

General Instructions: Answers to this paper must be written on the paper provided separately. You will **not** be allowed to write during the first 15 minutes (reading time). Attempt **all** questions from Section A and **any four** questions from Section B. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks. Mathematical tables and squared paper are provided.

SECTION A**40 Marks — Attempt ALL questions***Attempt all questions from this Section.*

- Q1.** (i) A shopkeeper buys an article whose marked price is Rs. 5000 at a discount of 10% and sells it to a customer at the marked price. If the rate of GST is 18%, find the GST paid by the shopkeeper to the Central Government. [3]
- (ii) Mr. Sharma deposits Rs. 600 per month in a recurring deposit account for 2 years at 8% per annum simple interest. Find the maturity value of the account. [3]
- (iii) If $A = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 4 & 0 \end{bmatrix}$, find the product AB . [4]
- Q2.** (i) Solve the inequation $3x - 5 < 5x + 7 \leq 17$, where $x \in W$ (whole numbers). Represent the solution set on a number line. [3]
- (ii) Solve the quadratic equation $3x^2 - 5x - 1 = 0$ using the quadratic formula, giving your answer correct to two decimal places. [4]
- (iii) If $a : b = 3 : 5$, find the value of $(3a + 2b) : (5a - b)$. [3]
- Q3.** (i) Find the value of k if $(x - 2)$ is a factor of $x^3 + 2x^2 - kx + 10$. [3]
- (ii) The 4th term of an A.P. is 11 and the 8th term exceeds twice the 4th term by 5. Find the first term, the common difference, and the sum of the first 20 terms. [4]
- (iii) Find the number of terms in the G.P. $3, 6, 12, \dots, 384$. [3]
- Q4.** (i) The point $P(3, -4)$ is reflected in the x -axis to P' , and P' is then reflected in the origin to P'' . Write the coordinates of P' and P'' , and name the single transformation that maps P to P'' . [4]
- (ii) Find the coordinates of the point which divides the line segment joining $A(-2, 3)$ and $B(4, -3)$ internally in the ratio $2 : 1$. [3]
- (iii) Prove that $\frac{\sin A}{1 - \cos A} = \operatorname{cosec} A + \cot A$. [3]

SECTION B**40 Marks — Attempt any FOUR questions***Attempt any four questions from this Section.*

- Q5.** (i) Find the equation of the line passing through $(2, -3)$ and parallel to the line $3x - 2y = 5$. [3]
- (ii) Find the equation of the line perpendicular to $2x + 3y = 7$ and passing through the point $(1, 1)$. [3]
- (iii) A straight line passes through the points $A(-1, 3)$ and $B(4, -2)$. Find its slope and its equation, and determine whether the point $(2, 0)$ lies on it. [4]

- Q6.** (i) In $\triangle ABC$, $DE \parallel BC$ with D on AB and E on AC . If $AD = 2$ cm, $DB = 3$ cm and $AE = 2.4$ cm, find EC and AC . [4]
- (ii) The ratio of corresponding sides of two similar triangles is $3 : 5$. If the area of the smaller triangle is 27 cm^2 , find the area of the larger triangle. [3]
- (iii) The areas of two similar triangles are 81 cm^2 and 144 cm^2 . If a side of the smaller triangle is 4.5 cm, find the corresponding side of the larger triangle. [3]
- Q7.** (i) A chord AB subtends an angle of 100° at the centre O of a circle. Find the angle subtended by AB at a point on the major arc and at a point on the minor arc. [4]
- (ii) PT is a tangent drawn from an external point P to a circle with centre O . If $PT = 8$ cm and $OP = 10$ cm, find the radius of the circle. [3]
- (iii) Two chords AB and CD of a circle intersect internally at a point P . If $PA = 4$ cm, $PB = 6$ cm and $PC = 3$ cm, find PD . [3]
- Q8.** (i) A die is thrown once. Find the probability of getting (a) a prime number, (b) a number greater than 4. [3]
- (ii) An article is marked at Rs. 2000. A shopkeeper sells it to a consumer at the marked price. If the rate of GST is 12%, find the total amount paid by the consumer. [3]
- (iii) The sum of the squares of two consecutive natural numbers is 313. Form a quadratic equation and find the numbers. [4]
- Q9.** (i) Find the sum of the first 25 terms of the A.P. whose n th term is $a_n = 3 + 2n$. [3]
- (ii) The 3rd term of a G.P. is 12 and the 5th term is 48. Find the G.P. (taking the common ratio positive) and its 6th term. [3]
- (iii) How many terms of the A.P. $18, 16, 14, \dots$ are needed to give a sum of 78? Explain the double answer. [4]
- Q10.** (i) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, show that $A^2 - 5A = 2I$, where I is the 2×2 identity matrix. [4]
- (ii) If $(2x + 3) : (5x - 2) = 3 : 4$, find the value of x . [3]
- (iii) A person deposits Rs. 1000 per month in a recurring deposit account for 1 year and receives Rs. 12390 at maturity. Find the rate of interest per annum. [3]
- Q11.** (i) Prove that $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = 1 + \sec A \operatorname{cosec} A$. [4]
- (ii) The point $A(2, 3)$ is reflected in the y -axis to A' , and A' is reflected in the x -axis to A'' . Write the coordinates of A' and A'' . [3]
- (iii) In a cyclic quadrilateral $ABCD$, $\angle A = (2x + 4)^\circ$ and $\angle C = (x + 8)^\circ$. Find the value of x and hence $\angle A$. [3]

ANSWER KEY & MARKING SCHEME

SECTION A

Answers

- Q1.** (i) Purchase price = $5000 \times 0.9 = 4500$. GST collected on sale = 18% of 5000 = 900; input tax credit = 18% of 4500 = 810. GST paid to Central Government (on value addition) = $\frac{1}{2}(900 - 810) = \text{Rs. } 45$ as CGST (or Rs. 90 total GST to government). (ii) $n = 24$, $P = 600$. Interest = $P \cdot \frac{n(n+1)}{2 \times 12} \cdot \frac{r}{100} = 600 \cdot \frac{24 \times 25}{24} \cdot \frac{8}{100} = 600 \cdot 25 \cdot 0.08 = 1200$. Maturity = $600 \times 24 + 1200 = \text{Rs. } 15600$. (iii) $AB = \begin{bmatrix} 2 & -4 \\ 11 & 2 \end{bmatrix}$.
- Q2.** (i) $3x - 5 < 5x + 7 \Rightarrow x > -6$; $5x + 7 \leq 17 \Rightarrow x \leq 2$. So $-6 < x \leq 2$; with $x \in W$: solution set $\{0, 1, 2\}$. (ii) $x = \frac{5 \pm \sqrt{25 + 12}}{6} = \frac{5 \pm \sqrt{37}}{6} \Rightarrow x \approx 1.85$ or $x \approx -0.18$. (iii) Taking $a = 3, b = 5$: $(3a + 2b) : (5a - b) = (9 + 10) : (15 - 5) = 19 : 10$.
- Q3.** (i) $f(2) = 8 + 8 - 2k + 10 = 0 \Rightarrow 26 - 2k = 0 \Rightarrow k = 13$. (ii) $a + 3d = 11$ and $a + 7d = 27 \Rightarrow 4d = 16, d = 4, a = -1$. $S_{20} = \frac{20}{2}[2(-1) + 19(4)] = 10(74) = 740$. (iii) $3 \cdot 2^{n-1} = 384 \Rightarrow 2^{n-1} = 128 = 2^7 \Rightarrow n = 8$ terms.
- Q4.** (i) $P' = (3, 4)$; $P'' = (-3, -4)$; the single transformation is reflection in the y -axis. (ii) Point = $\left(\frac{2(4) + 1(-2)}{3}, \frac{2(-3) + 1(3)}{3}\right) = (2, -1)$. (iii) $\frac{\sin A}{1 - \cos A} \cdot \frac{1 + \cos A}{1 + \cos A} = \frac{\sin A(1 + \cos A)}{\sin^2 A} = \frac{1 + \cos A}{\sin A} = \text{cosec } A + \cot A$.

SECTION B

Answers

- Q5.** (i) Slope = $\frac{3}{2}$; $y + 3 = \frac{3}{2}(x - 2) \Rightarrow 3x - 2y - 12 = 0$. (ii) Slope of given line = $-\frac{2}{3}$, so required slope = $\frac{3}{2}$; $y - 1 = \frac{3}{2}(x - 1) \Rightarrow 3x - 2y - 1 = 0$. (iii) Slope = $\frac{-2 - 3}{4 + 1} = -1$; equation $y - 3 = -(x + 1) \Rightarrow x + y = 2$. At $(2, 0)$: $2 + 0 = 2$, so the point lies on the line.
- Q6.** (i) $\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{2}{3} = \frac{2.4}{EC} \Rightarrow EC = 3.6$ cm; $AC = AE + EC = 6$ cm. (ii) Ratio of areas = $3^2 : 5^2 = 9 : 25$; $\frac{27}{A} = \frac{9}{25} \Rightarrow A = 75 \text{ cm}^2$. (iii) Ratio of sides = $\sqrt{81} : \sqrt{144} = 9 : 12 = 3 : 4$; side = $4.5 \times \frac{4}{3} = 6$ cm.
- Q7.** (i) Angle at major arc = $\frac{1}{2}(100^\circ) = 50^\circ$; angle at minor arc = $\frac{1}{2}(360^\circ - 100^\circ) = 130^\circ$. (ii) Radius = $\sqrt{OP^2 - PT^2} = \sqrt{100 - 64} = 6$ cm. (iii) $PA \cdot PB = PC \cdot PD \Rightarrow 4 \times 6 = 3 \times PD \Rightarrow PD = 8$ cm.
- Q8.** (i) Prime $\{2, 3, 5\}$: $P = \frac{3}{6} = \frac{1}{2}$; greater than 4 $\{5, 6\}$: $P = \frac{2}{6} = \frac{1}{3}$. (ii) Amount = $2000 + 12\% \text{ of } 2000 = 2000 + 240 = \text{Rs. } 2240$. (iii) $n^2 + (n + 1)^2 = 313 \Rightarrow 2n^2 + 2n - 312 = 0 \Rightarrow n^2 + n - 156 = 0 \Rightarrow (n - 12)(n + 13) = 0 \Rightarrow n = 12$. Numbers are 12 and 13.
- Q9.** (i) $a_1 = 5, a_{25} = 3 + 50 = 53$; $S_{25} = \frac{25}{2}(5 + 53) = 25 \times 29 = 725$. (ii) $ar^2 = 12, ar^4 = 48 \Rightarrow r^2 = 4, r = 2, a = 3$. G.P.: 3, 6, 12, 24, ...; 6th term = $3 \cdot 2^5 = 96$. (iii) $S_n = \frac{n}{2}[36 + (n - 1)(-2)] = n(19 - n) = 78 \Rightarrow n^2 - 19n + 78 = 0 \Rightarrow (n - 6)(n - 13) = 0 \Rightarrow n = 6$ or 13 . Both give 78 because terms 7 to 13 add up to 0.
- Q10.** (i) $A^2 = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}, 5A = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$, so $A^2 - 5A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2I$. (ii) $4(2x + 3) = 3(5x - 2) \Rightarrow 8x + 12 = 15x - 6 \Rightarrow 7x = 18 \Rightarrow x = \frac{18}{7}$. (iii) Deposits = 12000, interest = 390. $I = P \cdot \frac{n(n+1)}{2 \times 12} \cdot \frac{r}{100} = 1000 \cdot \frac{12 \times 13}{24} \cdot \frac{r}{100} = 65r \Rightarrow 65r = 390 \Rightarrow r = 6\%$.

- Q11.** (i) $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$. Writing $\cot A = \frac{1}{\tan A}$ and simplifying gives
- $$\frac{\tan^2 A}{\tan A - 1} - \frac{1}{\tan A(\tan A - 1)} = \frac{\tan^3 A - 1}{\tan A(\tan A - 1)} = \frac{\tan^2 A + \tan A + 1}{\tan A} = \tan A + 1 + \cot A = 1 + \sec A \operatorname{cosec} A.$$
- (ii) $A' = (-2, 3)$; $A'' = (-2, -3)$. (iii) $(2x + 4) + (x + 8) = 180 \Rightarrow 3x + 12 = 180 \Rightarrow x = 56$; $\angle A = 2(56) + 4 = 116^\circ$.