

**MID-TERM EXAMINATION — Class 10 Mathematics (Set 2)**

**Chapters:** Reflection, Mid-point & Section Formula, Equation of a Line, Circles, Trigonometric Identities, Heights & Distances, Probability, Measures of Central Tendency, Cylinder Cone & Sphere

**Max Marks: 80**

Name: \_\_\_\_\_

**Time: 2½ Hours**

**General Instructions:** Answers to this paper must be written on the paper provided separately. You will **not** be allowed to write during the first 15 minutes (reading time). Attempt **all** questions from Section A and **any four** questions from Section B. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks. Mathematical tables and squared paper are provided.

**SECTION A****40 Marks — Attempt ALL questions***Attempt all questions from this Section.*

- Q1.** (i) The point  $B(-2, 4)$  is reflected in the  $y$ -axis to  $B'$ , and  $B'$  is reflected in the  $x$ -axis to  $B''$ . Write the coordinates of  $B'$  and  $B''$ , and name the single transformation that maps  $B$  to  $B''$ . [4]
- (ii) Find the coordinates of the point that divides the line segment joining  $A(1, -3)$  and  $B(4, 3)$  internally in the ratio  $2 : 1$ . [3]
- (iii) Find the equation of the line passing through  $(2, -1)$  with slope  $-3$ . [3]
- Q2.** (i) A chord  $AB$  of a circle subtends an angle of  $130^\circ$  at the centre  $O$ . Find the angle subtended by  $AB$  at a point on the major arc. [4]
- (ii) Prove that  $(\sec \theta + \tan \theta)(1 - \sin \theta) = \cos \theta$ . [3]
- (iii) The length of the shadow of a vertical pole is  $6\sqrt{3}$  m when the height of the pole is 6 m. Find the angle of elevation of the sun. [3]
- Q3.** (i) A card is drawn at random from a well-shuffled pack of 52 playing cards. Find the probability that it is (a) a face card, (b) a spade. [3]
- (ii) Find the mean and the median of the data: 20, 22, 25, 28, 30. [4]
- (iii) Find the curved surface area of a solid cylinder of radius 7 cm and height 20 cm. (Take  $\pi = \frac{22}{7}$ .) [3]
- Q4.** (i) Find the volume of a sphere of radius 21 cm. (Take  $\pi = \frac{22}{7}$ .) [4]
- (ii) Find the coordinates of the mid-point of the line segment joining  $A(6, -4)$  and  $B(2, 8)$ . [3]
- (iii) Prove that  $(1 + \cot^2 A) \sin^2 A = 1$ . [3]

**SECTION B****40 Marks — Attempt any FOUR questions***Attempt any four questions from this Section.*

- Q5.** (i) Find the equation of the line with slope 2 passing through the point  $(-1, 3)$ . [3]  
(ii) Find the equation of the line passing through the points  $(0, 4)$  and  $(2, 0)$ . [3]  
(iii) Find the equation of the line through  $(3, 2)$  perpendicular to the line  $x + y = 6$ , and state its  $y$ -intercept. [4]
- Q6.** (i) From an external point  $P$ , a tangent  $PT$  is drawn to a circle with centre  $O$  and radius 5 cm. If  $OP = 13$  cm, find  $PT$  and the perimeter of quadrilateral  $OTPT'$  (where  $PT'$  is the other tangent). [4]  
(ii) In a cyclic quadrilateral  $ABCD$ ,  $\angle B = 110^\circ$ . Find  $\angle D$ . [3]  
(iii) Two chords  $AB$  and  $CD$  of a circle intersect internally at  $P$ . If  $PA = 9$  cm,  $PB = 4$  cm and  $PC = 6$  cm, find  $PD$ . [3]
- Q7.** (i) The angles of elevation of the top of a tower from two points at distances 9 m and 16 m from the base, in the same straight line with it, are complementary. Prove that the height of the tower is  $\sqrt{9 \times 16}$  m, and hence find the height. [5]  
(ii) From the top of a tower 50 m high, the angles of depression of two cars standing on the same side of the tower are  $45^\circ$  and  $30^\circ$ . Find the distance between the two cars (in surd form). [5]
- Q8.** (i) Find the mean of the following frequency distribution: values 5, 10, 15, 20, 25 with frequencies 4, 6, 10, 6, 4 respectively. [5]  
(ii) Find the median of the data: 7, 9, 11, 13, 15, 17, 19. [3]  
(iii) Find the mode of the data: 2, 3, 3, 4, 5, 3, 6, 4. [2]
- Q9.** (i) A solid cone of base radius 3 cm and height 4 cm is mounted on a solid hemisphere of radius 3 cm. Find the total volume of the solid (leave the answer in terms of  $\pi$ ). [4]  
(ii) Find the curved surface area of a cone of base radius 7 cm and slant height 25 cm. (Take  $\pi = \frac{22}{7}$ .) [3]  
(iii) Find the surface area of a sphere of radius 14 cm. (Take  $\pi = \frac{22}{7}$ .) [3]
- Q10.** (i) The point  $P(-3, 4)$  is reflected in the origin to  $P'$ , and  $P$  is reflected in the  $x$ -axis to  $P''$ . Write the coordinates of  $P'$  and  $P''$ , and find the area of  $\triangle PP'P''$ . [4]  
(ii) Find the ratio in which the  $x$ -axis divides the line segment joining  $A(2, -3)$  and  $B(5, 6)$ . [3]  
(iii) Find the equation of the line through  $(1, 1)$  perpendicular to the line  $2x - 3y = 6$ . [3]
- Q11.** (i) Prove that  $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$ . [4]  
(ii) Prove that  $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{1 + \sin \theta}{\cos \theta}$ . [3]  
(iii) Two dice are thrown together once. Find the probability of getting (a) a sum of 7, (b) a sum of at least 10. [3]

## ANSWER KEY & MARKING SCHEME

### SECTION A

Answers

- Q1.** (i)  $B' = (2, 4)$ ;  $B'' = (2, -4)$ ; single transformation is reflection in the origin. (ii) Point  
 $= \left( \frac{2(4) + 1(1)}{3}, \frac{2(3) + 1(-3)}{3} \right) = \left( \frac{9}{3}, \frac{3}{3} \right) = (3, 1)$ . (iii)  $y - (-1) = -3(x - 2) \Rightarrow 3x + y - 5 = 0$ .
- Q2.** (i) Angle at the major arc  $= \frac{1}{2}(130^\circ) = 65^\circ$ . (ii)  
 $(\sec \theta + \tan \theta)(1 - \sin \theta) = \frac{(1 + \sin \theta)(1 - \sin \theta)}{\cos \theta} = \frac{1 - \sin^2 \theta}{\cos \theta} = \frac{\cos^2 \theta}{\cos \theta} = \cos \theta$ . (iii)  
 $\tan \theta = \frac{6}{6\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$ .
- Q3.** (i) Face cards:  $P = \frac{12}{52} = \frac{3}{13}$ ; spade:  $P = \frac{13}{52} = \frac{1}{4}$ . (ii) Mean  $= \frac{20 + 22 + 25 + 28 + 30}{5} = \frac{125}{5} = 25$ ;  
 median = 25. (iii) CSA  $= 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 20 = 880 \text{ cm}^2$ .
- Q4.** (i)  $V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 21^3 = \frac{4}{3} \times \frac{22}{7} \times 9261 = 38808 \text{ cm}^3$ . (ii) Mid-point  
 $= \left( \frac{6+2}{2}, \frac{-4+8}{2} \right) = (4, 2)$ . (iii)  $(1 + \cot^2 A) \sin^2 A = \text{cosec}^2 A \cdot \sin^2 A = 1$ .

### SECTION B

Answers

- Q5.** (i)  $y - 3 = 2(x + 1) \Rightarrow 2x - y + 5 = 0$ . (ii) Slope  $= \frac{0 - 4}{2 - 0} = -2$ ;  $y = -2x + 4$ , i.e.  $2x + y - 4 = 0$ . (iii)  
 Slope of  $x + y = 6$  is  $-1$ , so required slope  $= 1$ ;  $y - 2 = 1(x - 3) \Rightarrow x - y - 1 = 0$ ;  $y$ -intercept  $= -1$ .
- Q6.** (i)  $PT = \sqrt{OP^2 - r^2} = \sqrt{169 - 25} = 12 \text{ cm}$ ; perimeter  $= 2(5 + 12) = 34 \text{ cm}$ . (ii)  $\angle D = 180^\circ - 110^\circ = 70^\circ$   
 . (iii)  $9 \times 4 = 6 \times PD \Rightarrow PD = 6 \text{ cm}$ .
- Q7.** (i) Let height  $= h$  and the two angles be  $\theta$  and  $90^\circ - \theta$ . Then  $\tan \theta = \frac{h}{9}$  and  $\tan(90^\circ - \theta) = \cot \theta = \frac{h}{16}$ .  
 Multiplying:  $\tan \theta \cot \theta = \frac{h^2}{9 \times 16} = 1 \Rightarrow h^2 = 144 \Rightarrow h = 12 \text{ m}$ . (ii) Nearer car:  $\tan 45^\circ = \frac{50}{d_1} \Rightarrow d_1 = 50$ ;  
 farther car:  $\tan 30^\circ = \frac{50}{d_2} \Rightarrow d_2 = 50\sqrt{3}$ . Distance  $= 50\sqrt{3} - 50 = 50(\sqrt{3} - 1) \text{ m} \approx 36.6 \text{ m}$ .
- Q8.** (i)  $\sum fx = 20 + 60 + 150 + 120 + 100 = 450$ ,  $\sum f = 30$ ; mean  $= \frac{450}{30} = 15$ . (ii)  $n = 7$ ; median = 4th value  
 $= 13$ . (iii) Mode  $= 3$ .
- Q9.** (i) Cone  $= \frac{1}{3}\pi(3)^2(4) = 12\pi$ ; hemisphere  $= \frac{2}{3}\pi(3)^3 = 18\pi$ ; total  $= 30\pi \text{ cm}^3$ . (ii) CSA  
 $= \pi rl = \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2$ . (iii) Surface area  $= 4\pi r^2 = 4 \times \frac{22}{7} \times 196 = 2464 \text{ cm}^2$ .
- Q10.** (i)  $P' = (3, -4)$ ;  $P'' = (-3, -4)$ . In  $\triangle PP'P''$ :  $PP' = 8$  (vertical) and horizontal distance from  $P''$  to line  
 $x = -3$  is 6; area  $= \frac{1}{2} \times 8 \times 6 = 24 \text{ sq units}$ . (ii) On the  $x$ -axis  $y = 0$ ; ratio  $k : 1$ :  $\frac{6k - 3}{k + 1} = 0 \Rightarrow k = \frac{1}{2}$ ; ratio  
 $= 1 : 2$ . (iii) Slope of given line  $= \frac{2}{3}$ , so required slope  $= -\frac{3}{2}$ ;  $y - 1 = -\frac{3}{2}(x - 1) \Rightarrow 3x + 2y - 5 = 0$ .

**Q11.** (i)  $(\operatorname{cosec} A - \sin A) = \frac{\cos^2 A}{\sin A}$ ,  $(\sec A - \cos A) = \frac{\sin^2 A}{\cos A}$ ,  $(\tan A + \cot A) = \frac{1}{\sin A \cos A}$ . Their product

$$= \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} \cdot \frac{1}{\sin A \cos A} = 1.$$

(ii) Multiplying numerator and denominator suitably and using  $\sec^2 \theta - \tan^2 \theta = 1$  gives  $\frac{1 + \sin \theta}{\cos \theta}$ . (iii) Sum 7:  $P = \frac{6}{36} = \frac{1}{6}$ ; sum  $\geq 10$ :  $(4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6), P = \frac{6}{36} = \frac{1}{6}$ .